Parametrized Automata

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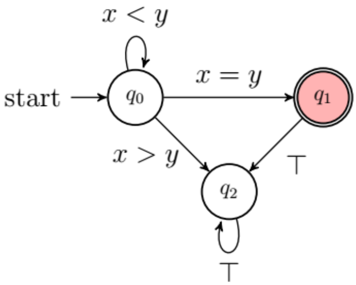
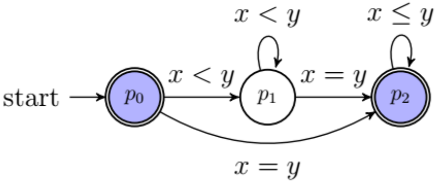
AVM 2024





Motivation

- Finite-State Automata (FSA): Regular languages over finite alphabets
- What if the alphabet is infinite? (E.g., arrays using real numbers...)
- Lots and lots of extensions of FSA for infinite alphabets
- In this presentation: Quick, example-driven introduction to Parametrized Automata (PA) [1], especially determinism and complementation
- Not in this presentation: Implementation

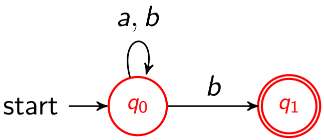
[1] See Parametrized Automata over Infinite Alphabets: Properties and Complementation, Franziska Alber, Master thesis, 2024 (coming soon!)

Finite-State Automata

An FSA $A = (D, Q, q_0, \delta, F)$ is a tuple of:

- D: finite, non-empty alphabet
- Q: finite set of states
- $q_0 \in Q$: initial state
- $\delta \subseteq Q \times D \times Q$: transition relation
- $F \subseteq Q$: set of accepting states

Alphabet: $D = \{a, b\}$, examples of words over D: $ab, babbba, \ldots$



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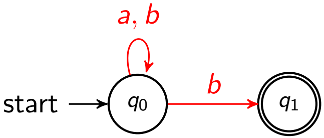

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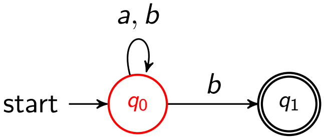

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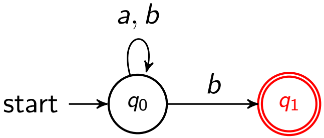
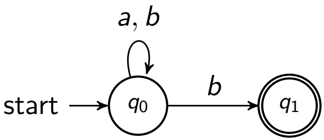
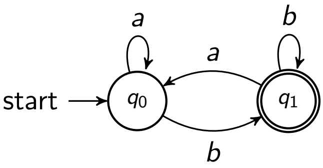


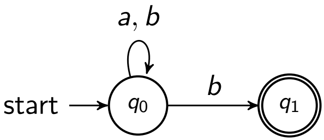


(a) A_1



(b) A_2

Determinism and Complementation



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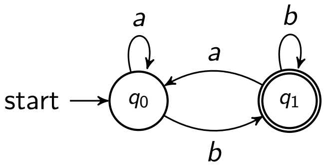


(b) A_2

Determinism:

- transition function
- exactly one possible path/run for every word
- in every state, exactly one exiting transition for every letter

Determinism leads to easy complementation!

Towards Infinite Alphabets

- Register Automata/Finite-Memory Automata [19]
- Symbolic Automata [12]
- Variable Automata [16]
- Parametrized Automata [18]
- Symbolic Register Automata [10]
- Register Set Automata [17]
- Extended Symbolic Finite Automata [11]
- Fresh-Variable Automata [3]
- Guarded Variable Automata over Infinite Alphabets [4]
- Register Automata with Linear Arithmetic [9]

Second Slide to Prove a Point

- Single-Use Register Automata [8]
- Pebble Automata [21]
- Usage Automata [2]
- Parametric Semilinear Data Automata [15]
- Parikh Automata [20]
- Timed Automata [14]
- Deterministic Memory Automata over Ordered Data[5]
- Data Automata and Two-Variable First-Order Logic [6]
- Alternating 1-Register Automata and LTL with Freeze Quantifiers [13]
- Nominal Automata [7]
- Streaming Data-String Acceptors [1]

Meet the Parents

Symbolic Automata use logical formulas ("guards") instead of letters. Properties are similar to FSA.

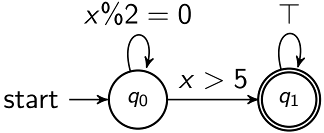


Figure: A symbolic automaton.

Variable Automata compare input letters to non-reassignable variables $y_1, y_2, \ldots, y_k$. The symbol z represents all letters not assigned to a variable.

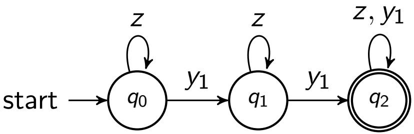
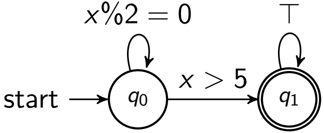


Figure: A variable automaton.

Meet the Parents

Symbolic Automata use logical formulas ("guards") instead of letters. Properties are similar to FSA.



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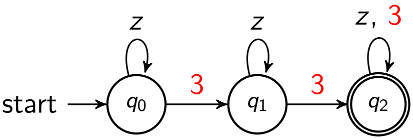


$\mu(y_1) = 3$

Parametrized Automata

A PA $A = (M, Q, q_0, \delta, F)$ is a tuple of:

- $M = (D, I)$: structure where D is an infinite alphabet
- Q: finite set of states
- $q_0 \in Q$: initial state
- $\delta \subseteq Q \times \Phi \times Q$: transition relation ($\Phi$: set of formulas)
- $F \subseteq Q$: set of accepting states

x: placeholder for current letter, y: parameter

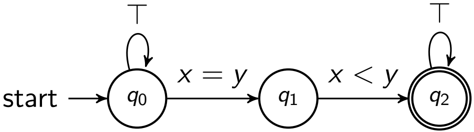


Figure: P_1 accepts all unsorted words and cannot be complemented.

Parametrized Automata are not closed under complementation! (Proof at the end)

Determinism per Assignment

- "Local" approach: Exactly one possible exiting transition for every state, letter and parameter assignment
- (+) Always works: Algorithms for Symbolic Automata are applicable
- (−) Does not imply that every word can take exactly one path ⇒ not sufficient for complementation!

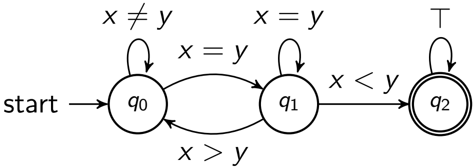


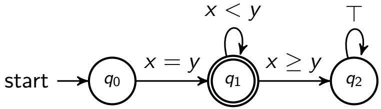
Figure: P_1': equivalent to P_1 and deterministic per assignment

Example: Word $w = (3, 2)$

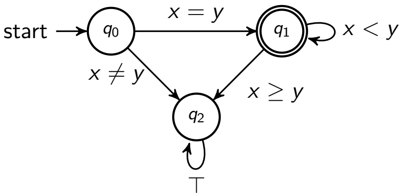
assignment	term. state
$\mu(y) = 3$	q_2
$\mu(y) = 0$	q_0
$\mu(y) = 2$	q_1

Strongly Deterministic Parametrized Automata (SDPA)

- "Global" approach: Exactly one possible path for every word
- (+) Easy complementation
- (−) Not every PA is equivalent to an SDPA (not even every complementable PA!)



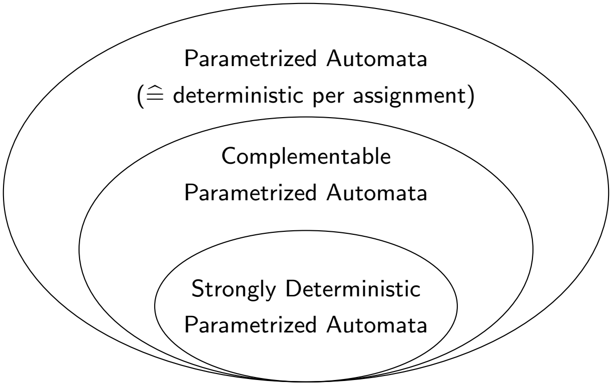
(a) Strongly deterministic



(b) Deterministic per assignment

Figure: Two different representations for the language of words in which the first letter is strictly largest.

Relations between the different subclasses of parametrized automata



Complementable PA $\subsetneq$ PA

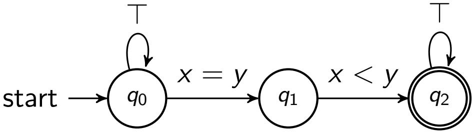
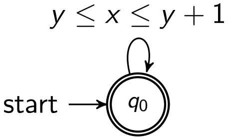


Figure: P_1 identifying all unsorted words.

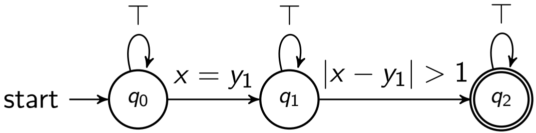
There is no PA identifying the complement language of $L(P_1)$:

- Complement PA of P_1 has to identify all sorted words (letters in ascending order)
- Assume P_1^c exists, n states $\Rightarrow$ the word $(1, 2, \ldots 3n)$ traverses some state thrice $\Rightarrow$ loop in section $(i, i+1, \ldots, i+k)$
- Word $(1, \ldots, i+k, i, \ldots, n)$ falsely accepted.

SDPA $\subsetneq$ Complementable PA



(a) A_3



(b) A_3^c

There is no SDPA A equivalent to A_3^c:

- In SDPA, the paths of prefixes of words are predetermined
- Consider sequence of words $(1, \frac{1}{2}, 2), (1, \frac{1}{2}, \frac{1}{3}, 1 + \frac{1}{2}), \ldots$
- Prove all paths of prefixes $(1, \frac{1}{2}, \ldots, \frac{1}{i})$ have to terminate in distinct states
- Number of states unbounded $\Rightarrow$ A does not exist.

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