

# PyQBF

A Python Framework for Solving Quantified Boolean Formulas



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# Introduction



# This is a QBF

## Example

$$\underbrace{\forall x_1 \exists x_2 \exists x_3 \forall x_4 \dots \exists x_n}_{\text{Prefix}} :$$

$$\begin{aligned} &(x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5) \wedge \\ &(\neg x_1 \vee x_4 \vee \neg x_9 \vee x_{11}) \wedge \\ &(\neg x_7 \vee \neg x_5 \vee \neg x_{10}) \wedge \end{aligned}$$

...

$$\underbrace{\hspace{10em}}_{\text{Propositional Formula}}$$

# This is a QBF

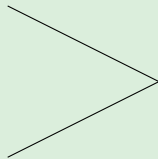
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**Two Parts**

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Boolean Variables

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## Example

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Propositional Formula

Quantifiers

# This is a QBF

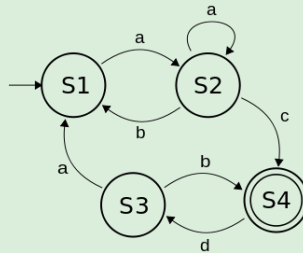
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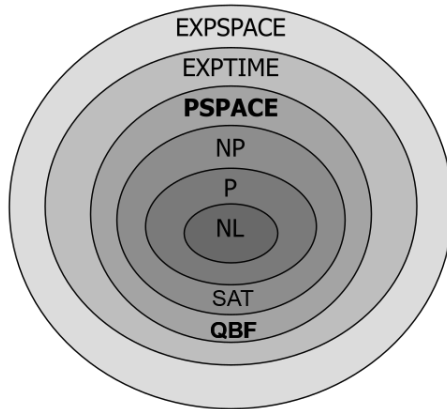
$(x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5) \wedge$   
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 $(\neg x_7 \vee \neg x_5 \vee \neg x_{10}) \wedge$   
 $\dots$

$\underbrace{\hspace{10em}}_{\text{Propositional Formula}}$

e.g. used in Bounded Model Checking



# QBF complexity



"Propositional logic +  $\forall$  and  $\exists$ "



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- Quantified Boolean Formulas (QBFs) are an important research topic
- Regularly we have the QBF-gallery with new formulas and solvers
- A lot of improvement can be seen

## **...with a little bit of rain**

- Only few QBF-Solver have an API

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- Only few QBF-Solver have an API
- Domain specific knowledge is necessary to use them
- Thus writing a prototype is hard

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PyQBF

# PySAT



# Not a new Concept

- In 2018, Ignatiev et al. released a paper<sup>1</sup> identifying the problems with SAT-solvers

<sup>1</sup>Ignatiev, A., Morgado, A., and Marques-Silva, J. (2018, June). PySAT: A Python toolkit for prototyping with SAT oracles. In International Conference on Theory and Applications of Satisfiability Testing (pp. 428-437). Cham: Springer International Publishing.

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- Low-level representation required is not sufficient for state-of-the-art problems
- They developed *PySAT*<sup>2</sup> — A toolkit for prototyping with SAT Oracles

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<sup>2</sup><https://pysathq.github.io/>

## Easy prototyping with SAT — Example

```
from pysat.formula import CNF
from pysat.solvers import Solver

formula = CNF(from_file="./my/cnf.dimacs")
formula2 = CNF(from_aiger="./your/aiger.aag")

with Solver(name='cadical153', bootstrap_with=formula) as solver:
    solver.add_clause([1,2,3])
    solver.append_formula(formula2)
    print(solver.solve())
```

# Many features

- Solving formulas
- Model enumeration
- Generation of cardinality constraints
- Preprocessing
- Certification
- ...



# PyQBF



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**Expanding the power of PySAT to the domain of QBFs!**

## Easy prototyping with QBF – Example

```
from pyqbf.formula import PCNF
from pyqbf.solver import Solver

formula = PCNF(from_file="./my/cnf.qdimacs")
formula2 = PCNF(from_aiger="./your/aiger.aag")
formula2.forall(1,2,3).exists(5,6,7,8)

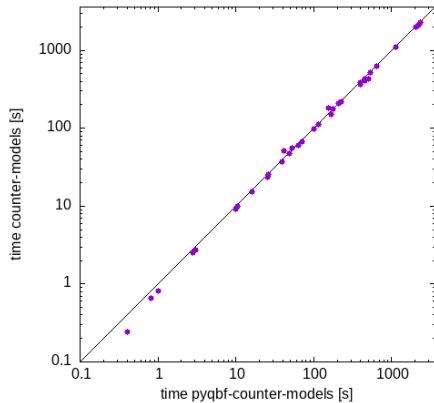
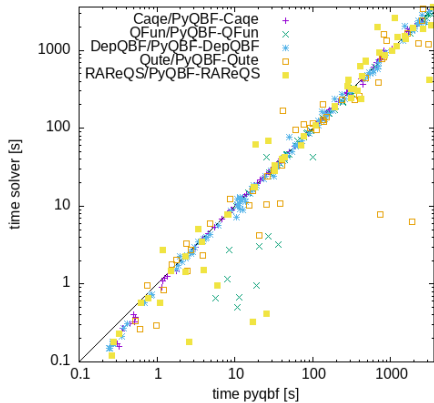
with Solver(name='depqbf', bootstrap_with=formula) as solver:
    solver.add_clause([1,2,3])
    solver.append_formula(formula2)
    print(solver.solve())
```

# Features

- Solving formulas with 5 state-of-the-art QBF-solvers
- (Counter-)Model enumeration
- Preprocessing with Bloqqer
- Import CNFs
- Normalize formulas
- Incremental solving with non-assuming solvers using QuAPI
- ...



# ...and all of that with only little overhead



# Availability

- *PyQBF: A Python Framework for Solving Quantified Boolean Formulas* was accepted for the 19th International Conference on Integrated Formal Methods (iFM24)
- We furthermore submitted an artifact publicly available at [10.5281/zenodo.11118824](https://zenodo.org/records/11118824)
- The current release version of PyQBF can be found at <https://gitlab.sai.jku.at/qbf/pyqbf>
- The documentation of the framework is available at <https://qbf.pages.sai.jku.at/pyqbf/>

# Thank you for your attention!



# Any Questions?



**JKU**

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