Efficient Manipulation of Logical Formulas as Decision Diagrams

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Supported by the **UNKP-23-3-I-BME-8** New National Excellence Program of the Ministry for Culture and Innovation from the source of the National Research, Development and Innovation Fund

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BME FACULTY OF ELECTRICAL ENGINEERING AND INFORMATICS

ftsrg Critical Systems Research Group

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- Heterogeneous system models
  - Mix of multiple high-level languages
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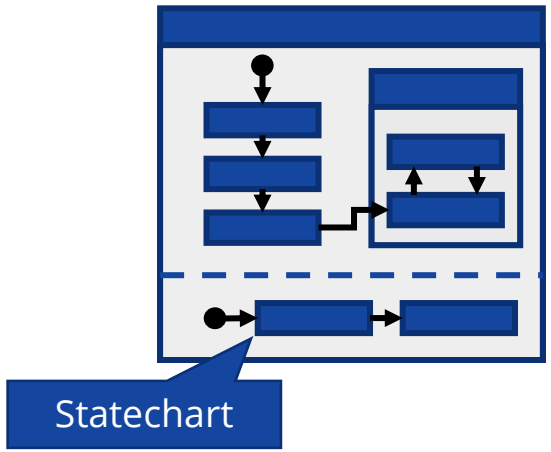

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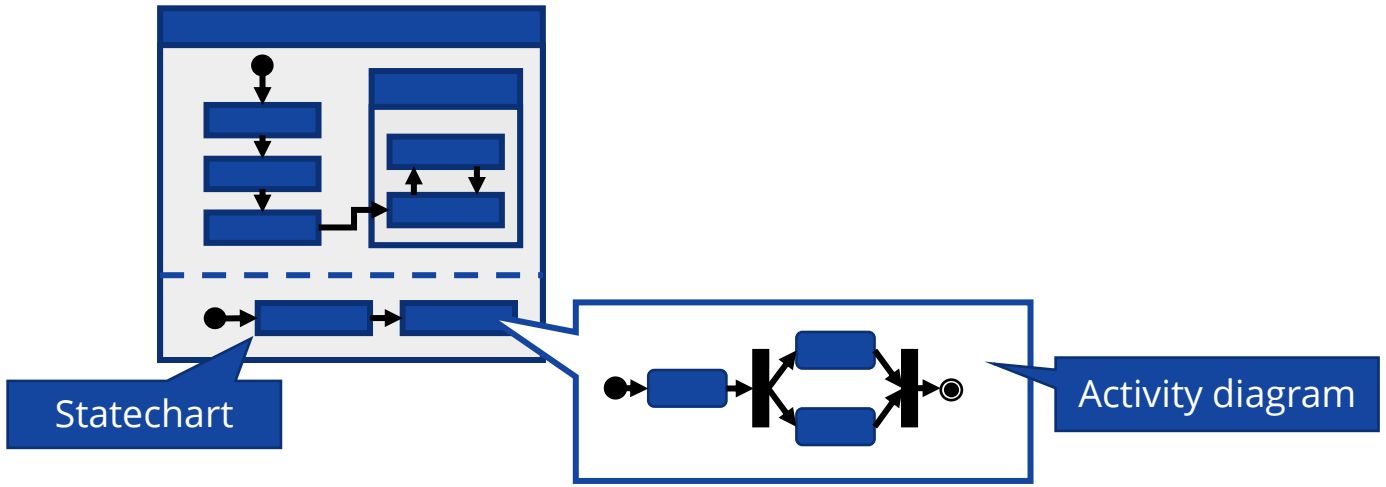

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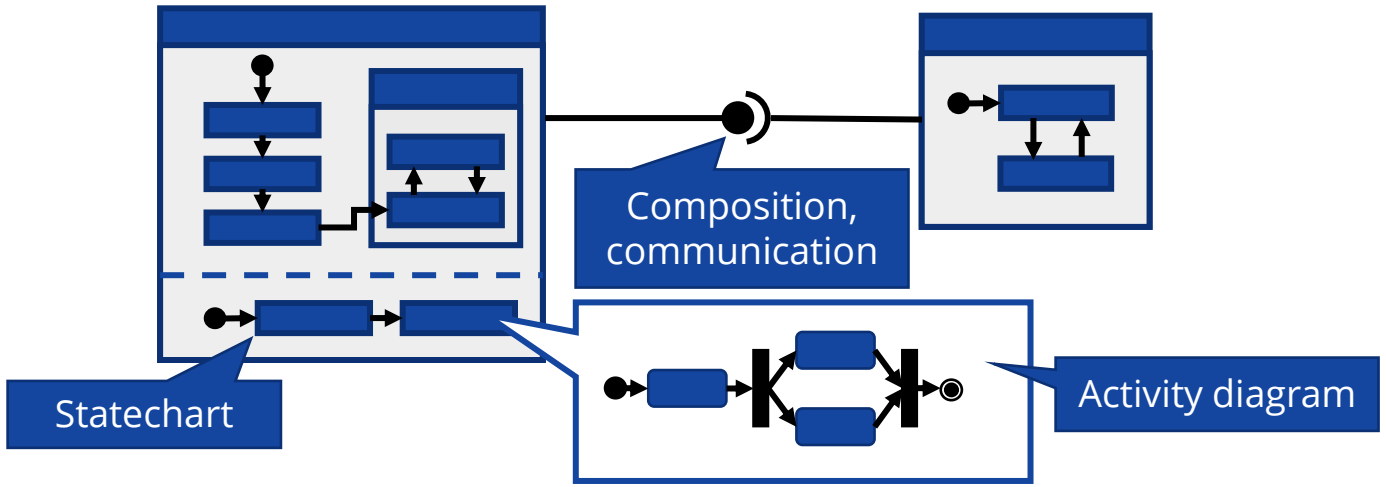

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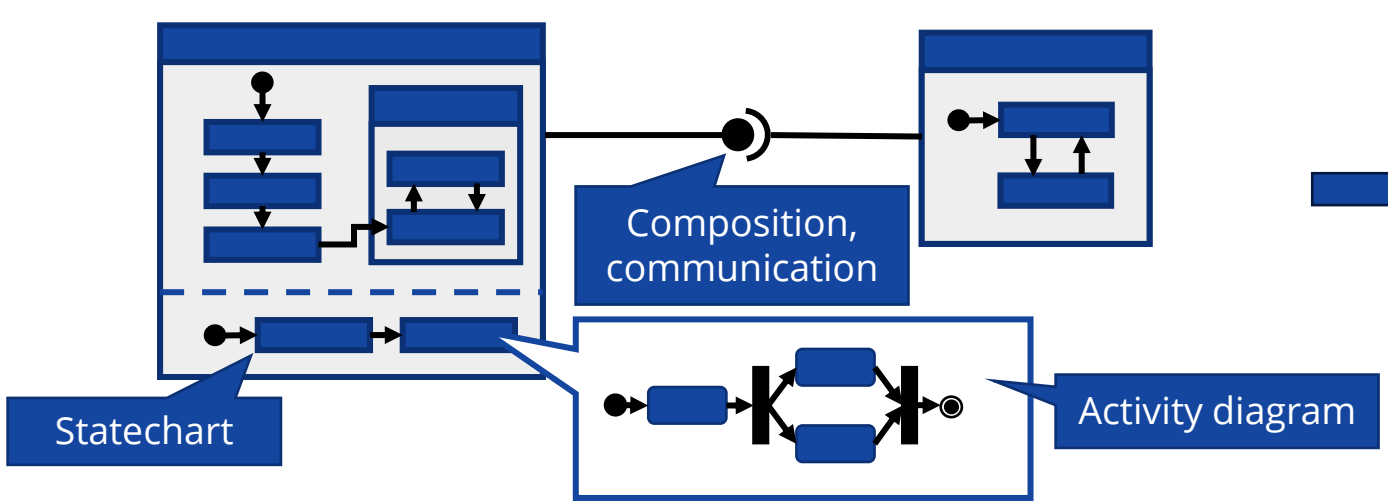

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I: x = 0

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T: (x < 5 ∧ x′ = x + 1) ∨ (x >= 5 ∧ x′ = x)

SMT formulas

→ Transformed to an SMT-based representation

Model-Based Testing of Asynchronously Communicating Distributed Controllers**, Bence Graics et al**, FACS 2023

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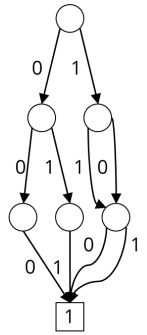
Extensions and generalization of the saturation algorithm in model checking, **Vince Molnár**, PhD Thesis, 2019



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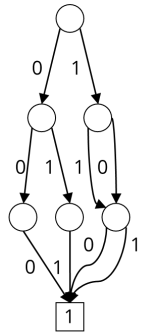
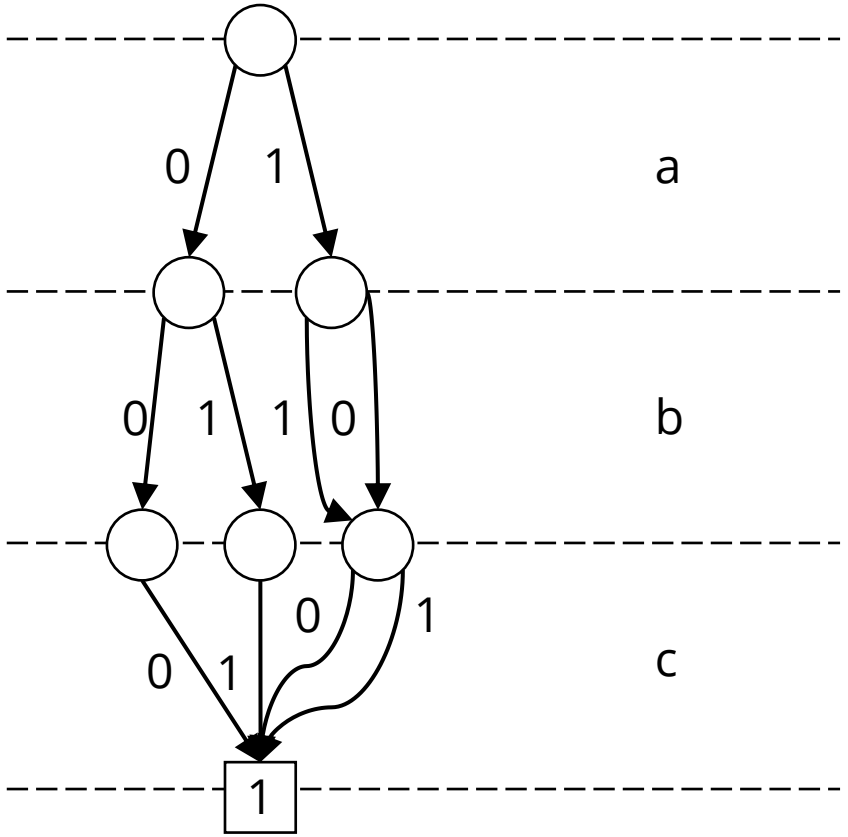


Goal: Exploit the advantages of **decision-diagram**-based algorithms (e.g., saturation) on **SMT-based** model representations

Decision diagrams

- Compact representation of a set of vectors



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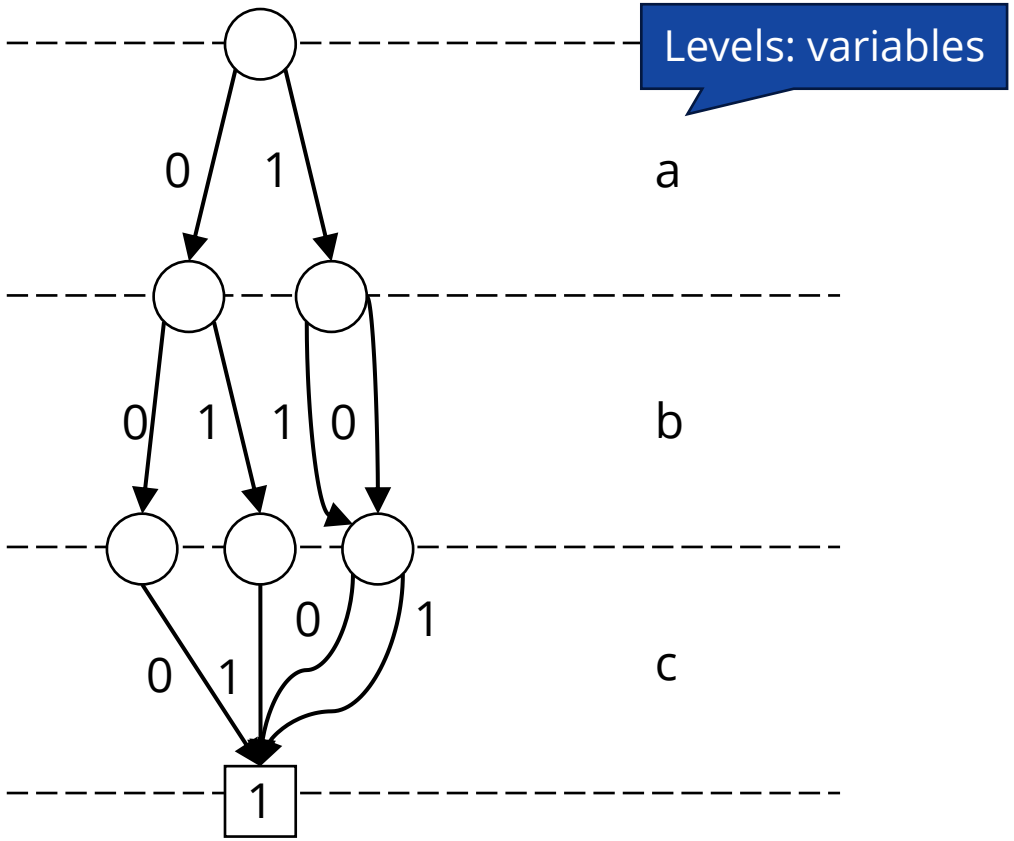

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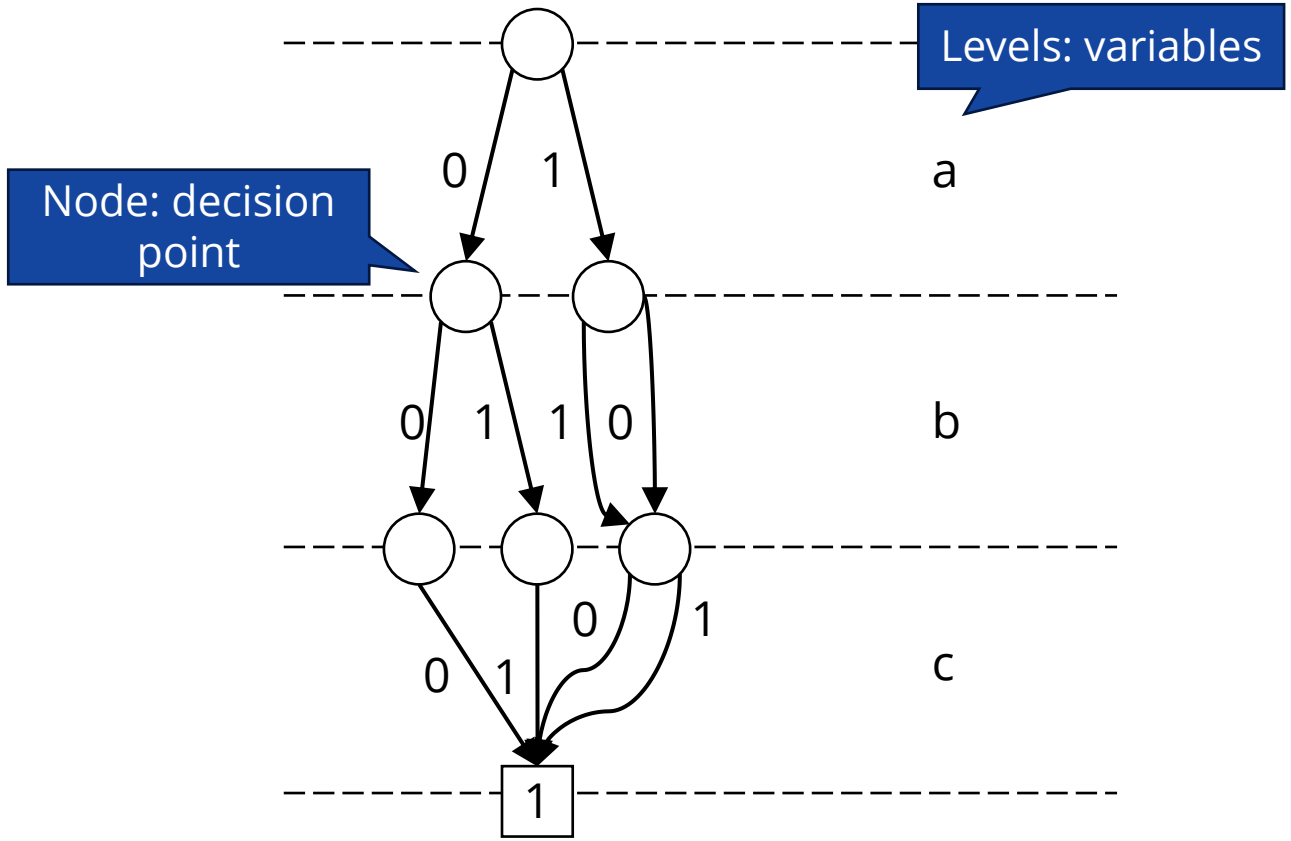

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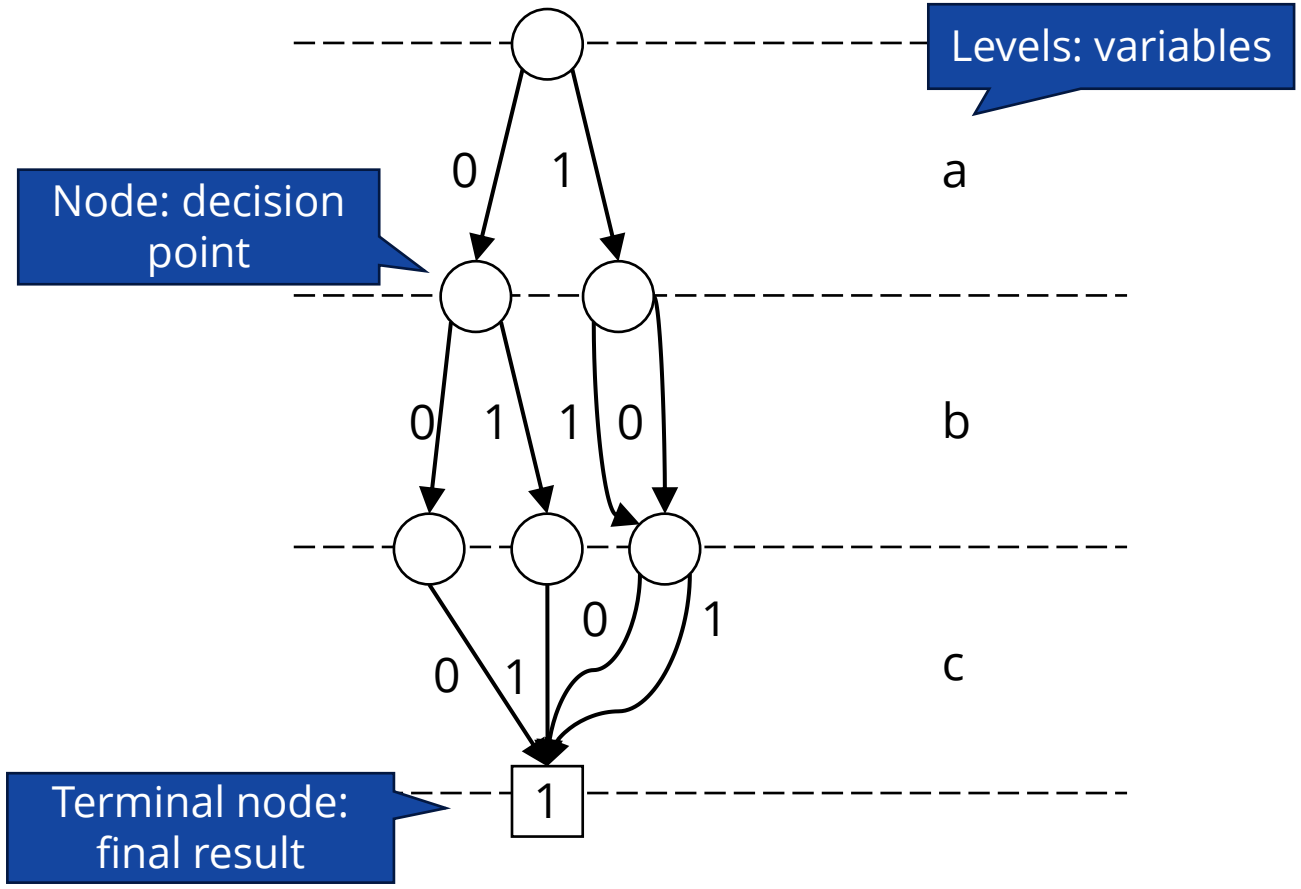

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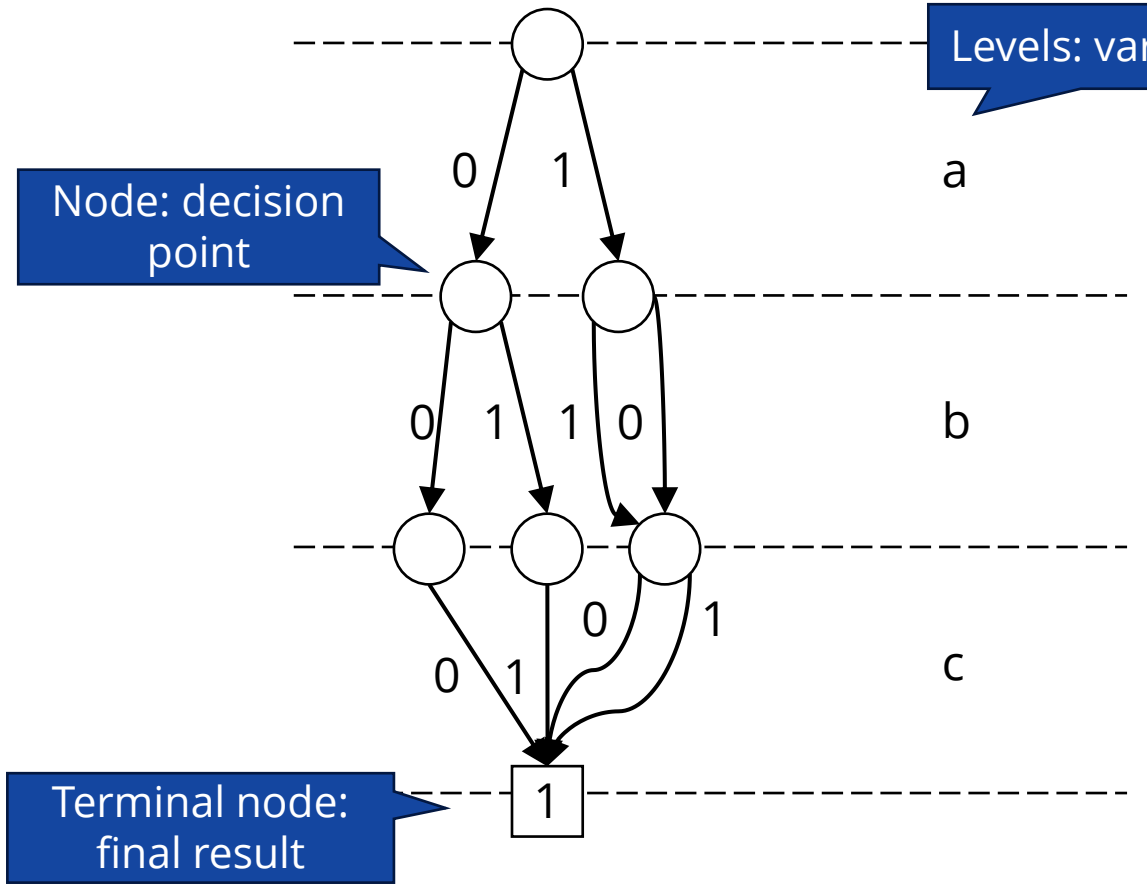

Usage for model checking:

- Encode states with k levels
- Encode transitions with 2k levels
- Model step: relational product

  → Calculate fixed point

level i: x
level i+1: x'

Decision diagram from SMT formula?

- One possible way: enumerate all solutions first

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SMT formula

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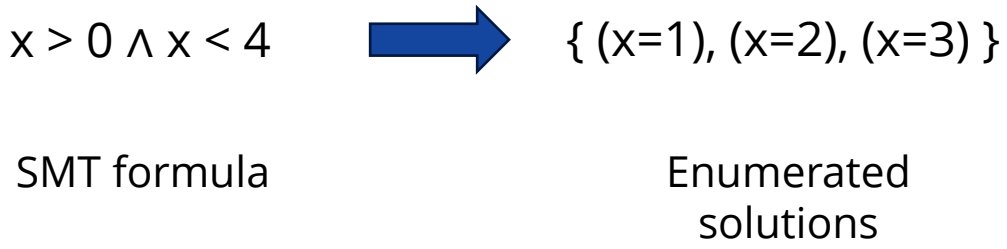

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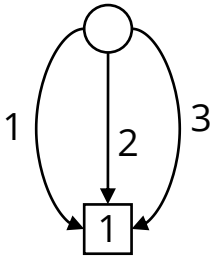

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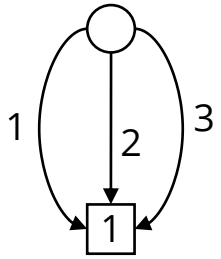

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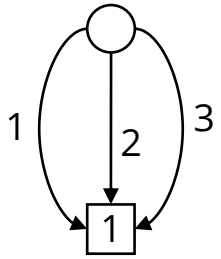

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How to represent **general** transition relations given as **SMT formulas**?

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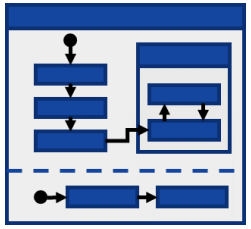

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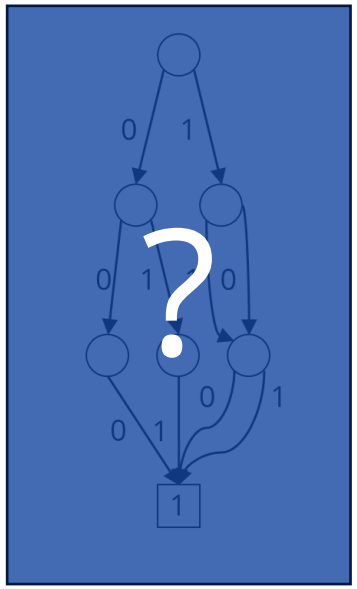

Decision diagrams

Symbolic model checker

✓ / X

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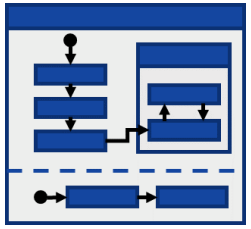

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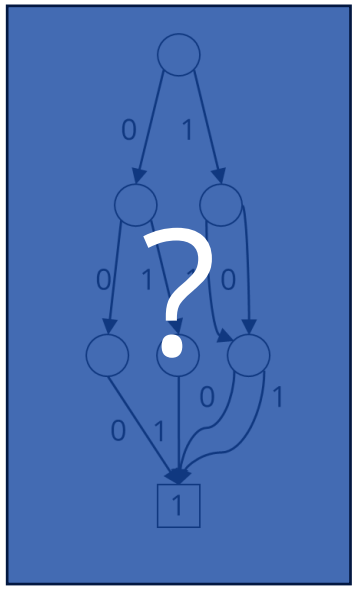

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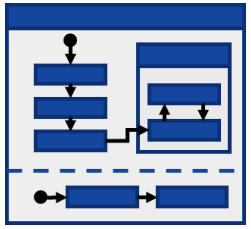

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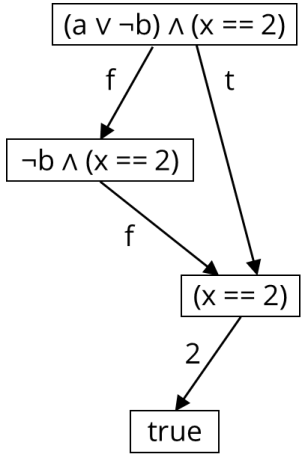

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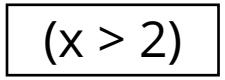

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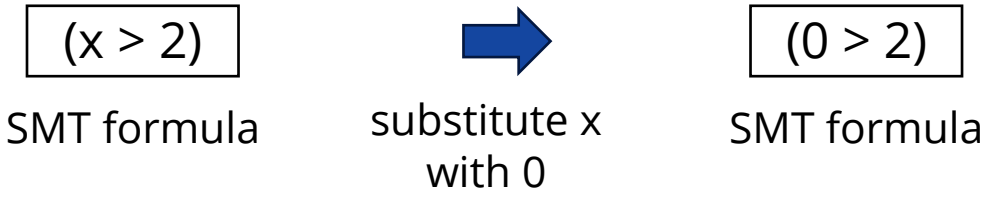

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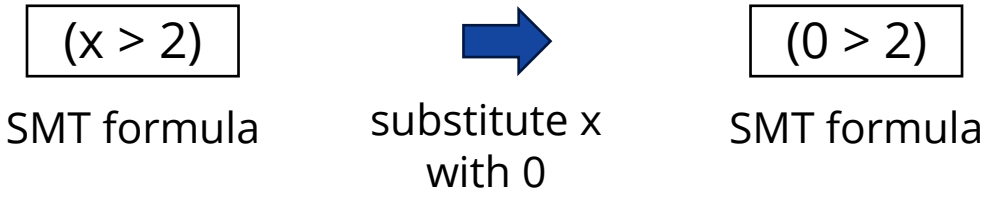


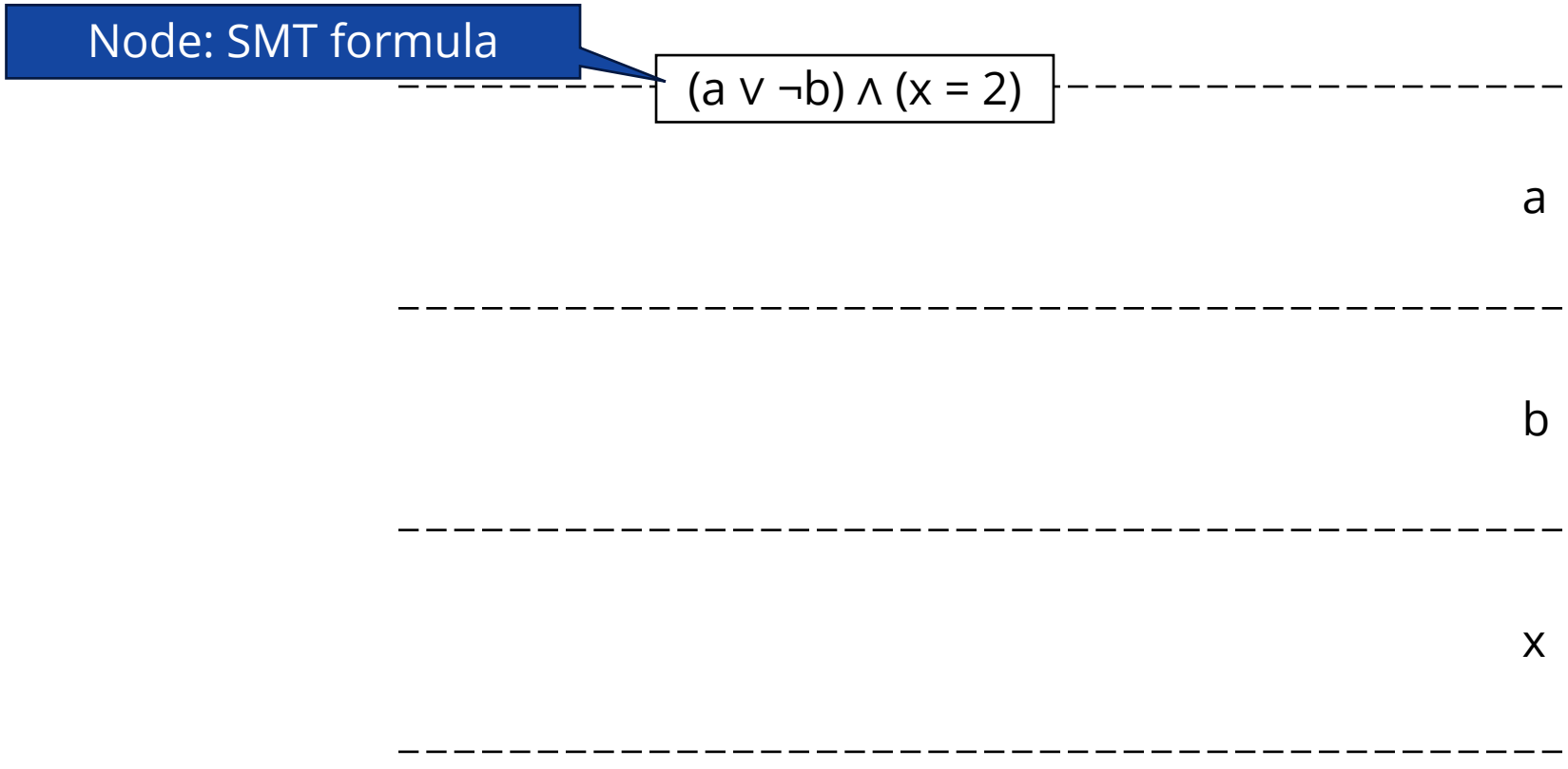
Observation: SMT formulas and the **variable substitution** operation span a structure that is similar to decision diagrams

SMT formulas → Nodes

Variable substitution → Edges

Controlling SAT/SMT solvers with decision diagrams to support abstraction-based model checking
Almási Nóra, BME VIK TDK 2020

Substitution diagram



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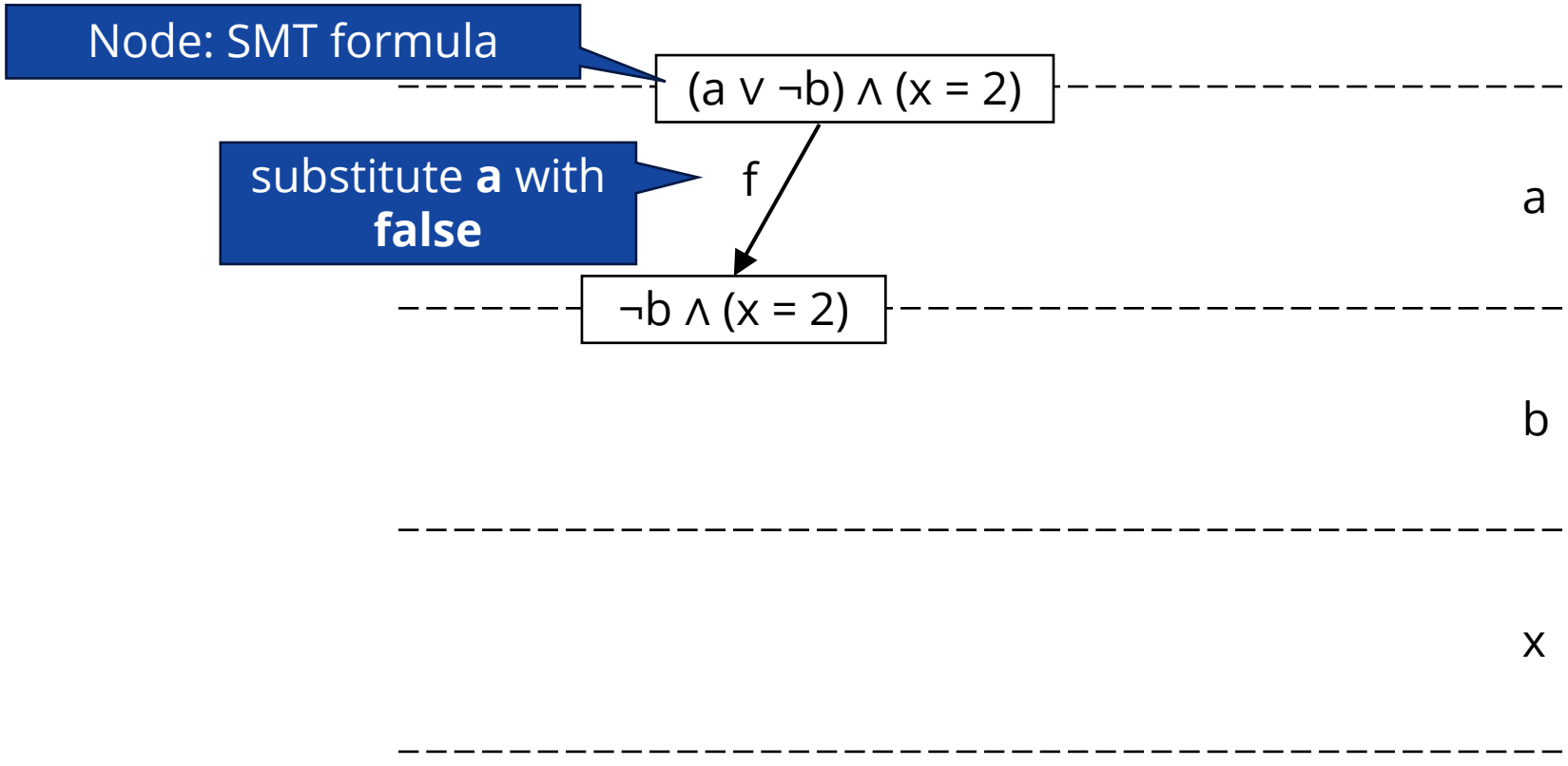

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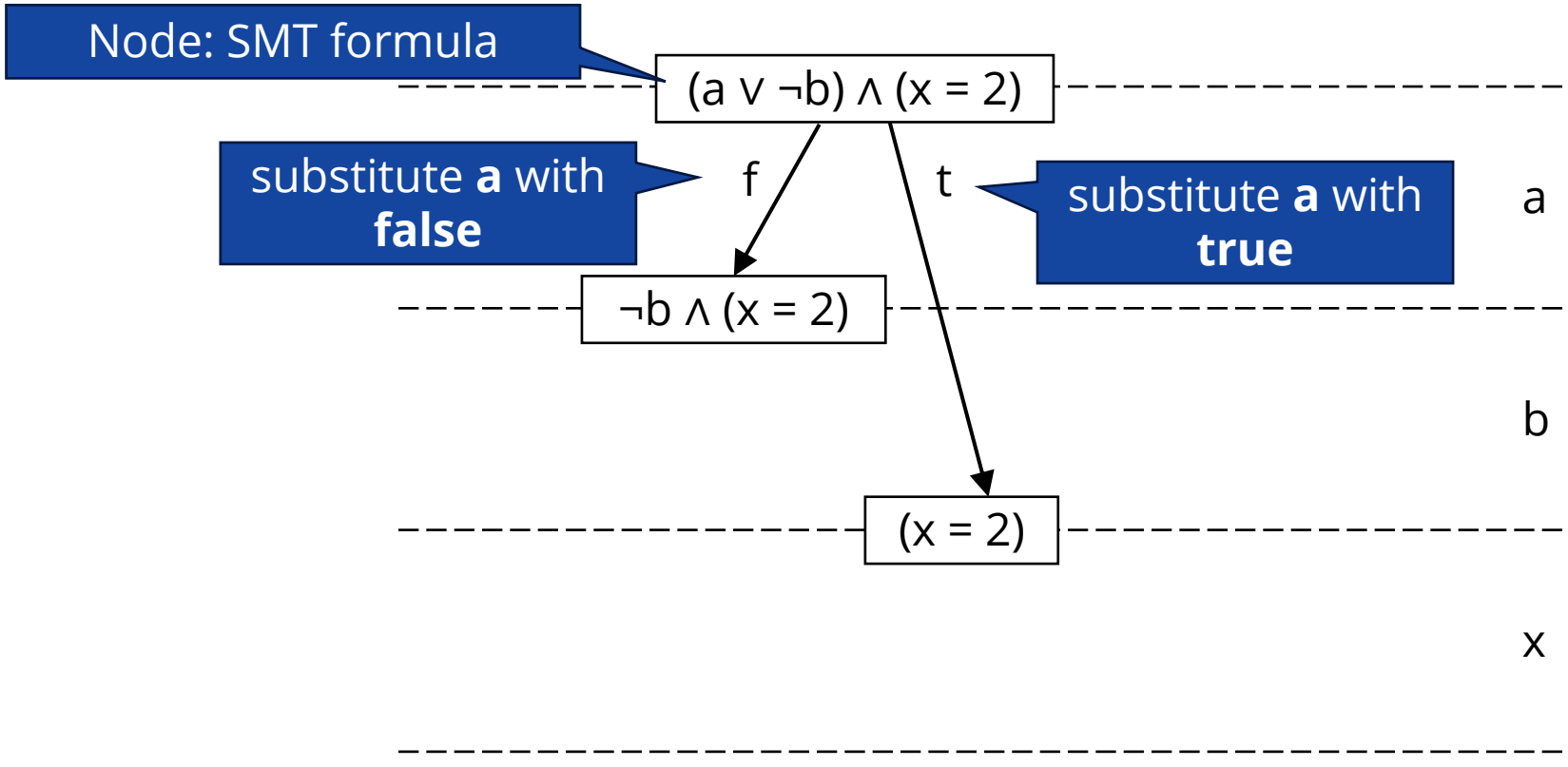

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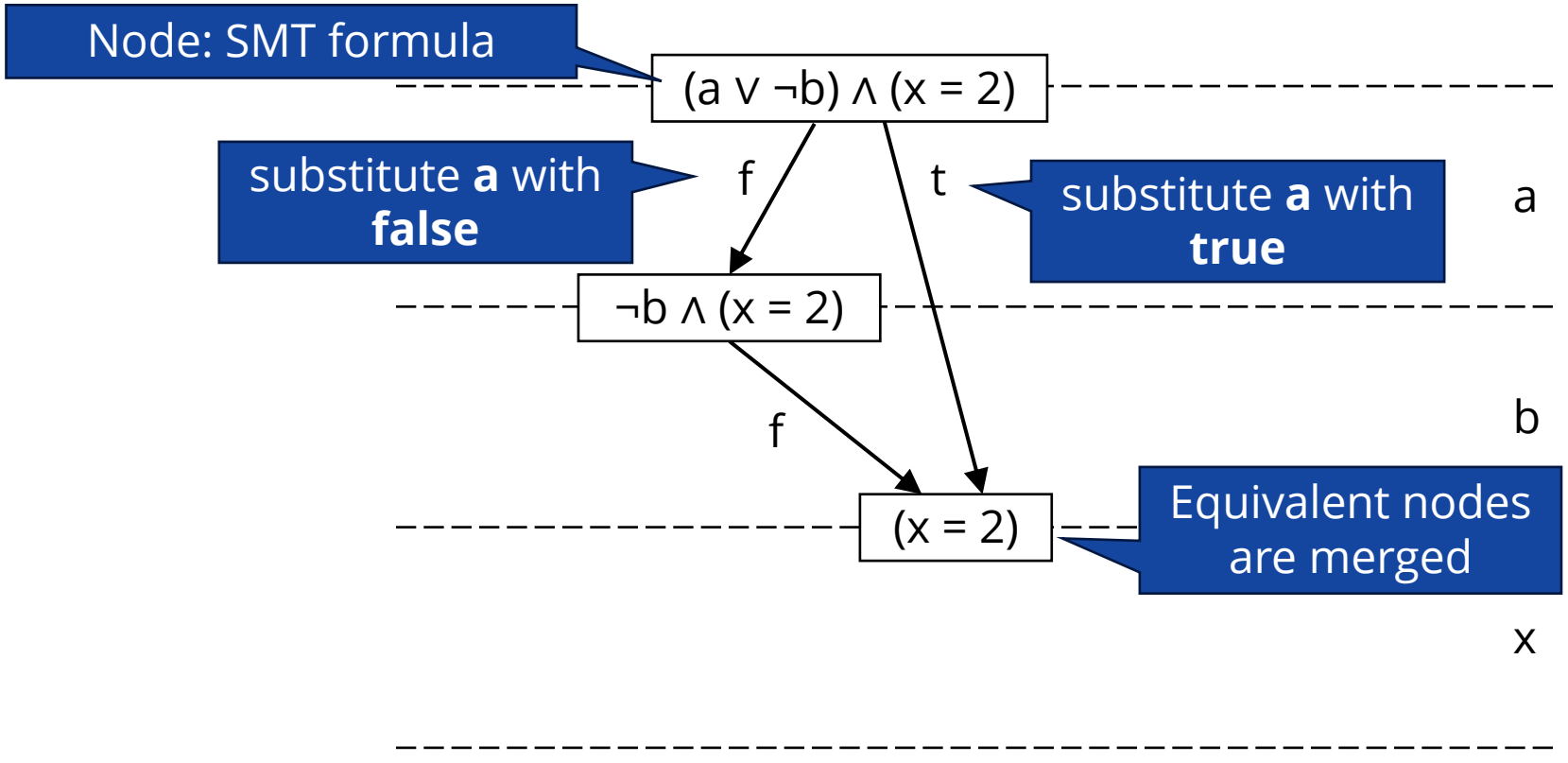

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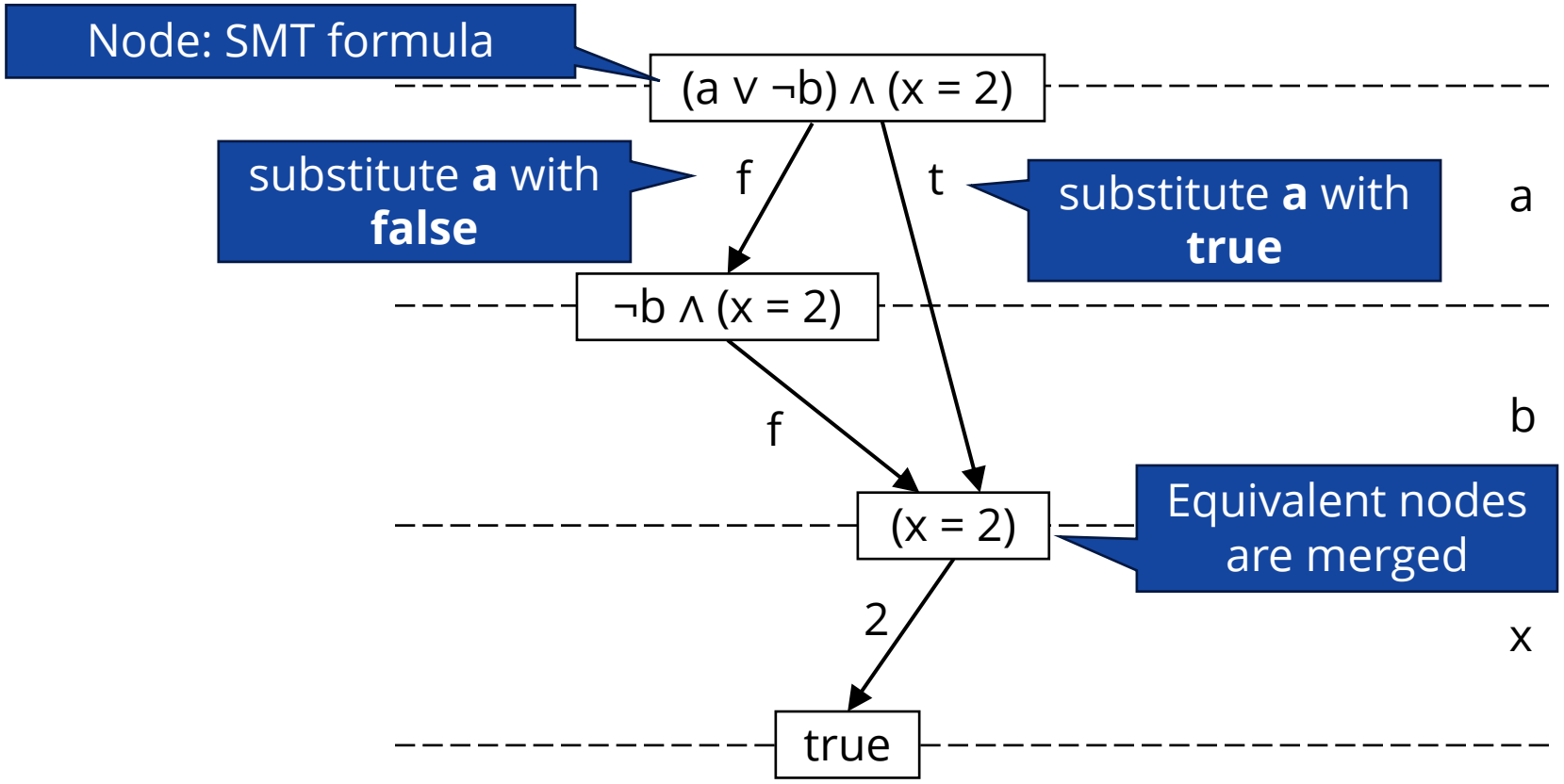

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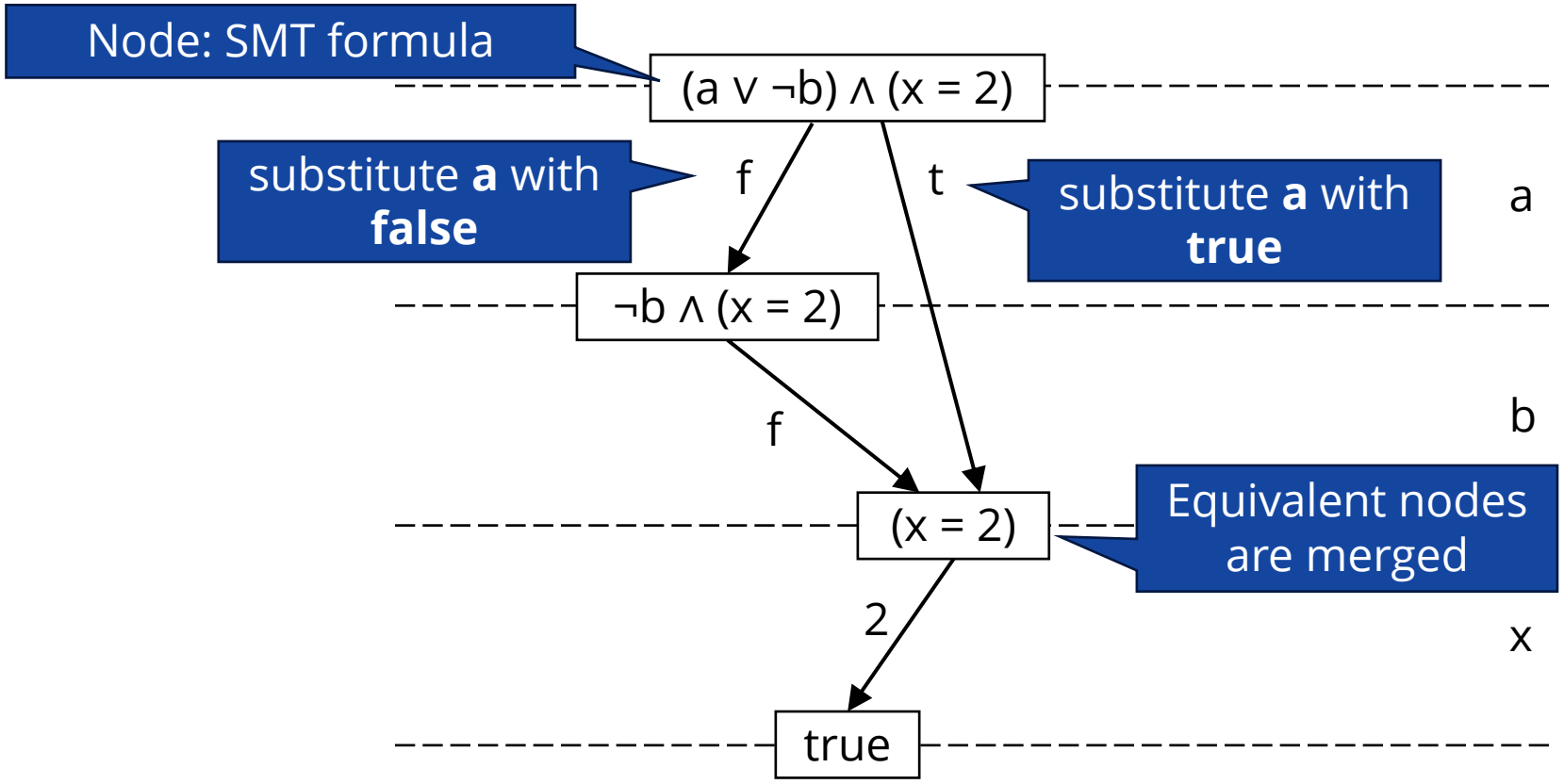

Lazy evaluation: presence of edges and children evaluated only when queried!

syntactically or with an **SMT-solver**

Model checking with substitution diagrams

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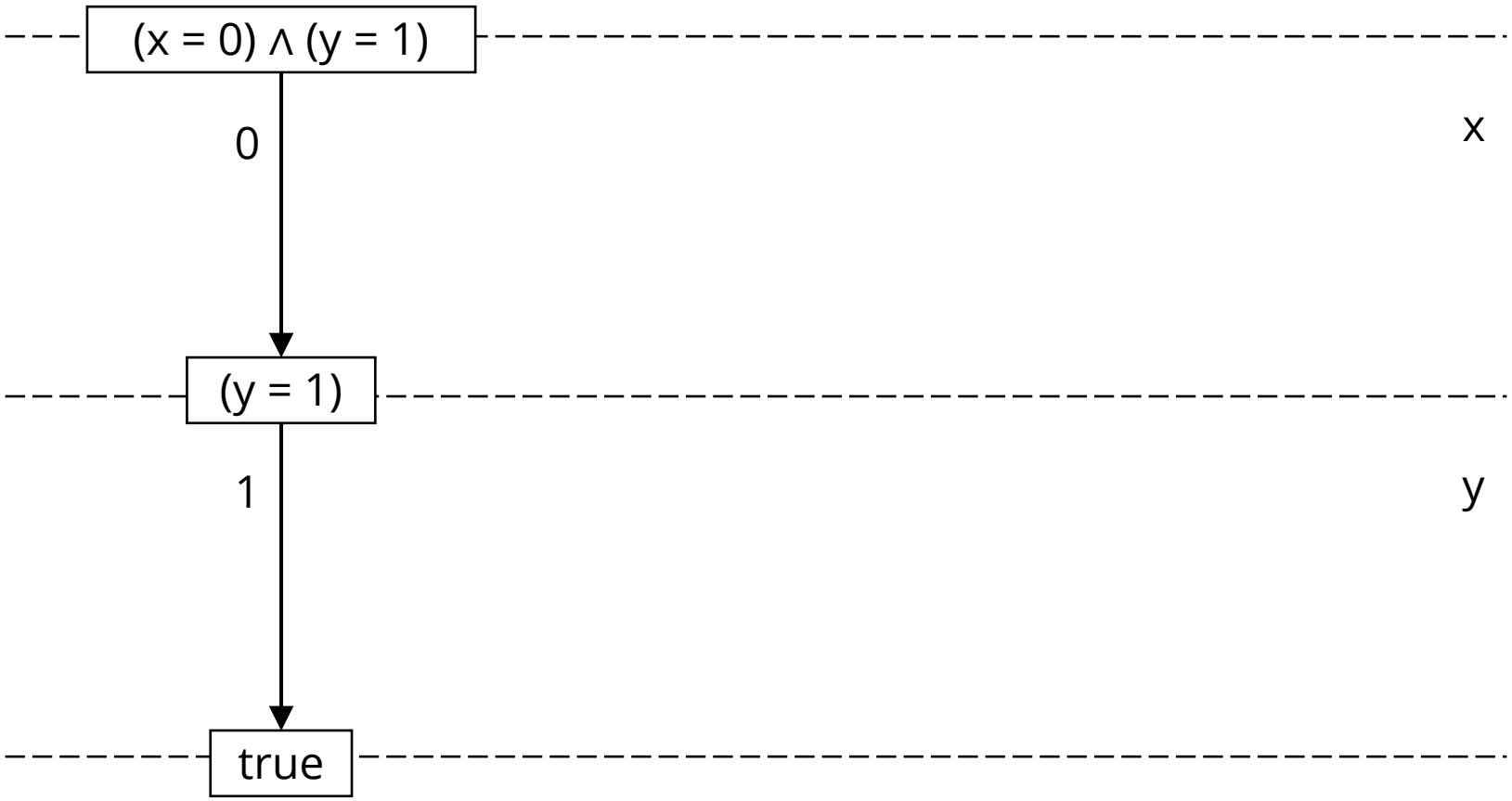

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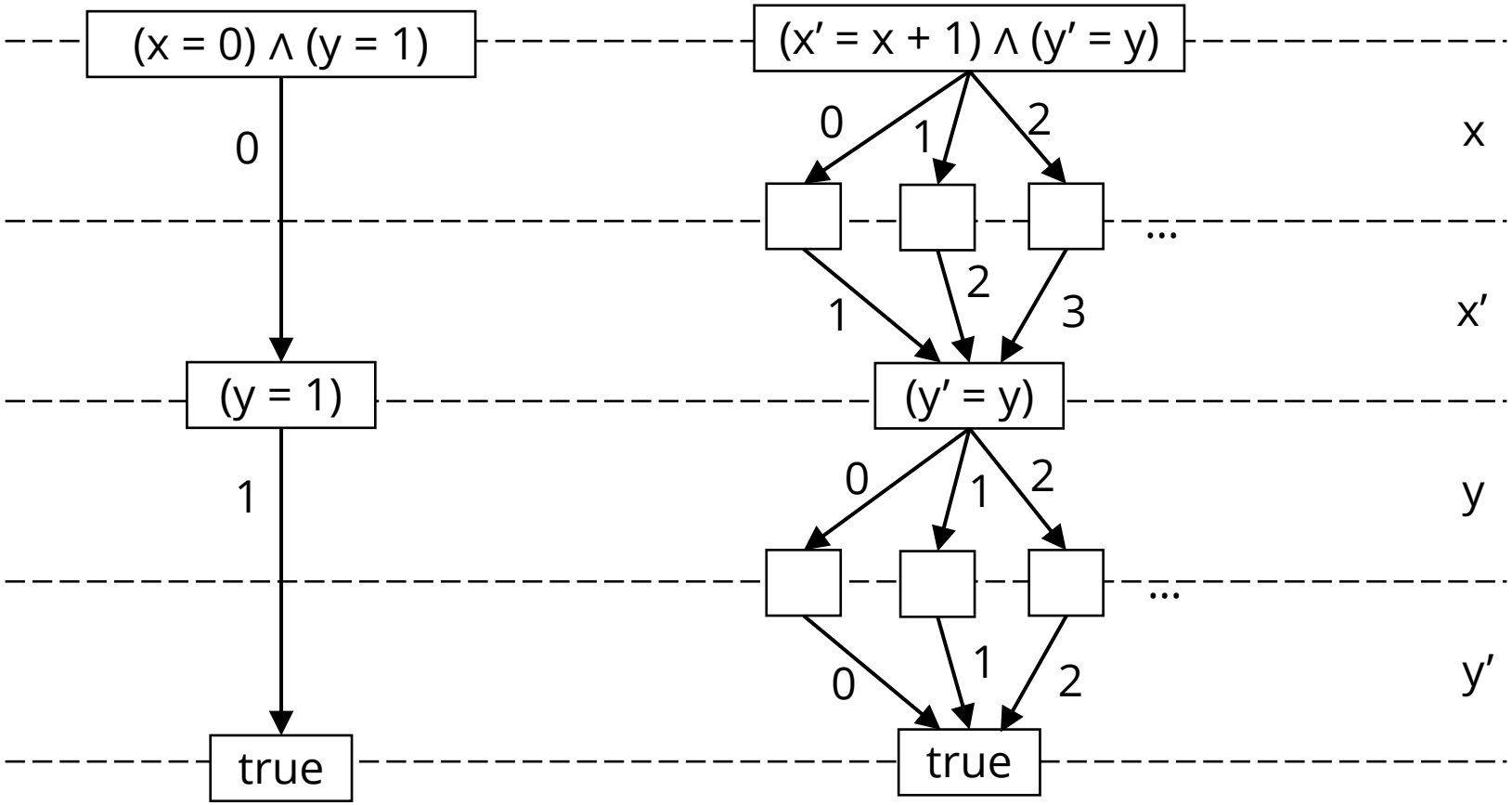
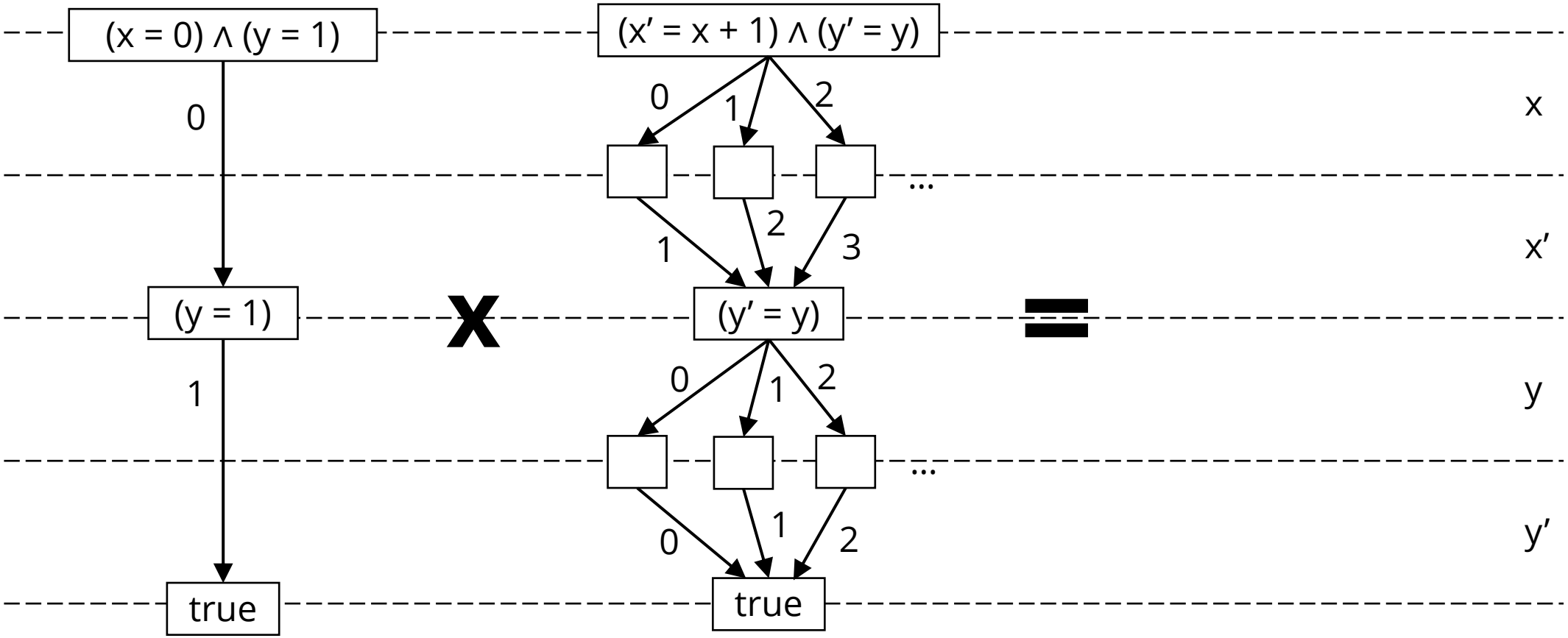


Relational product: model step



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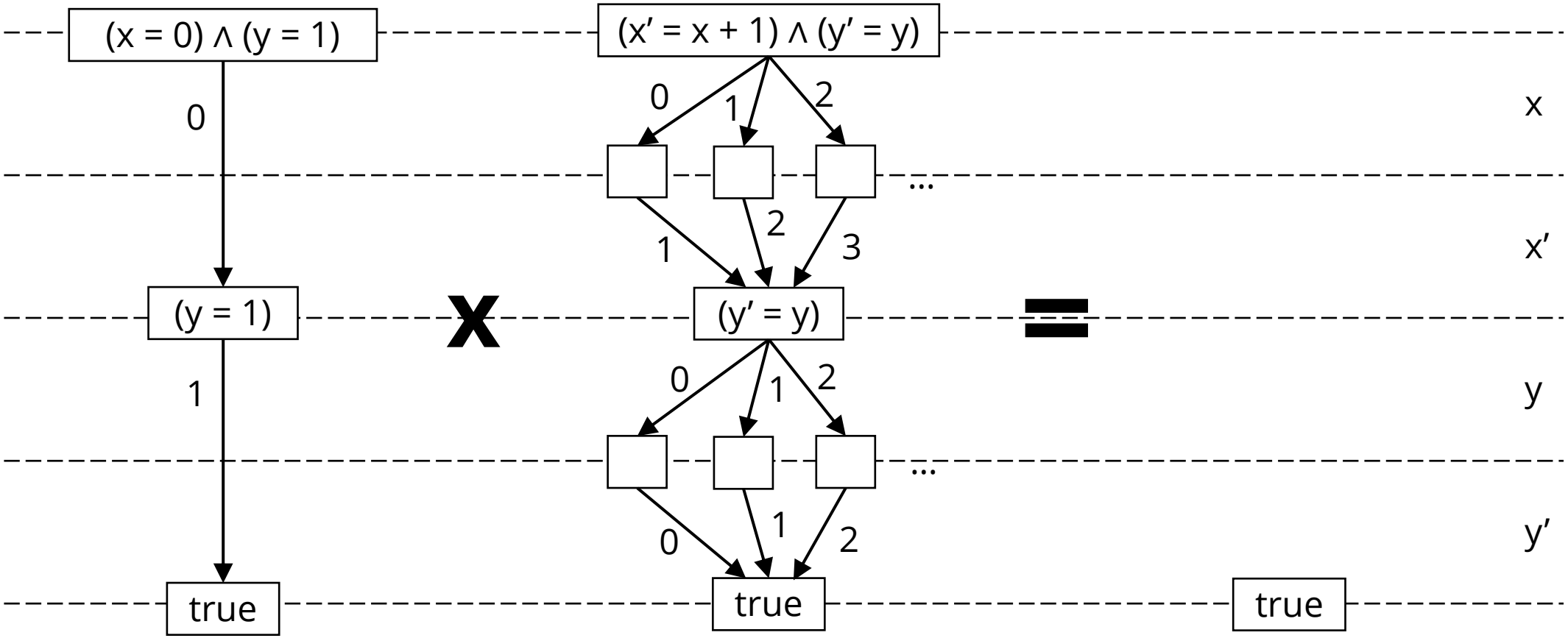

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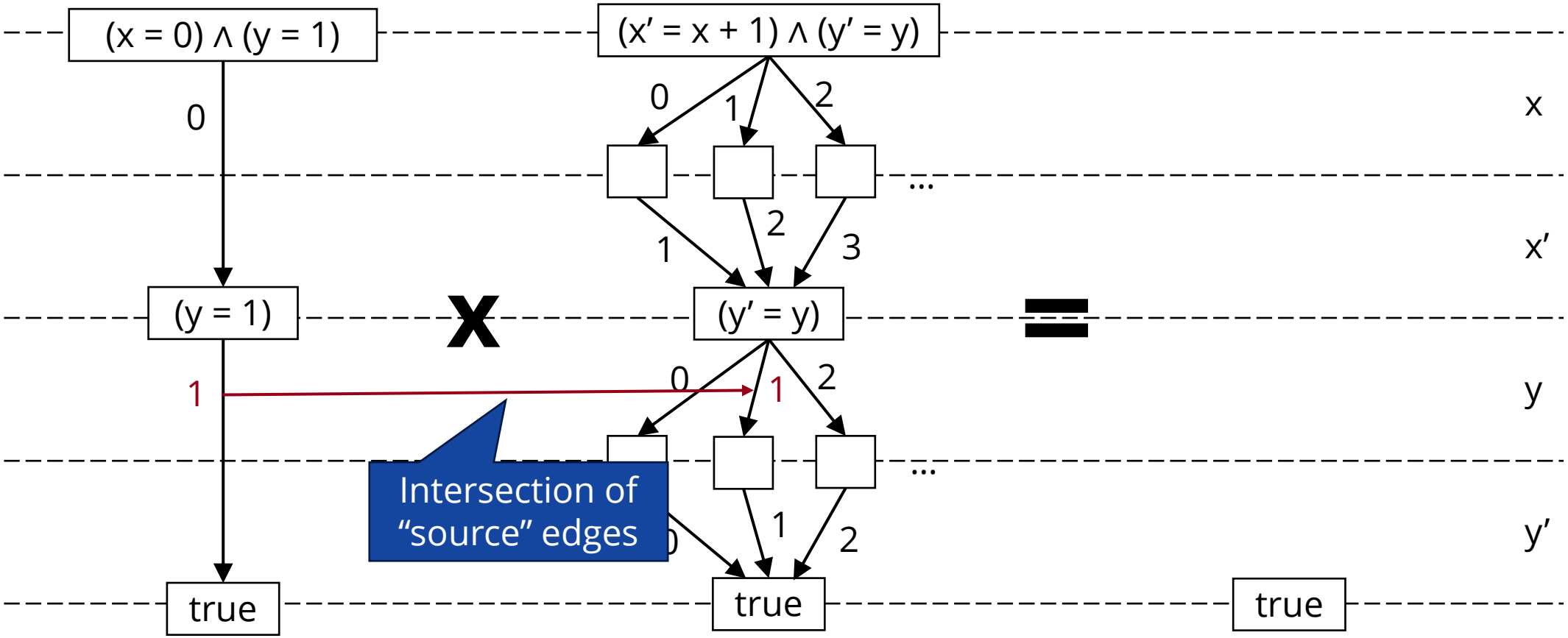

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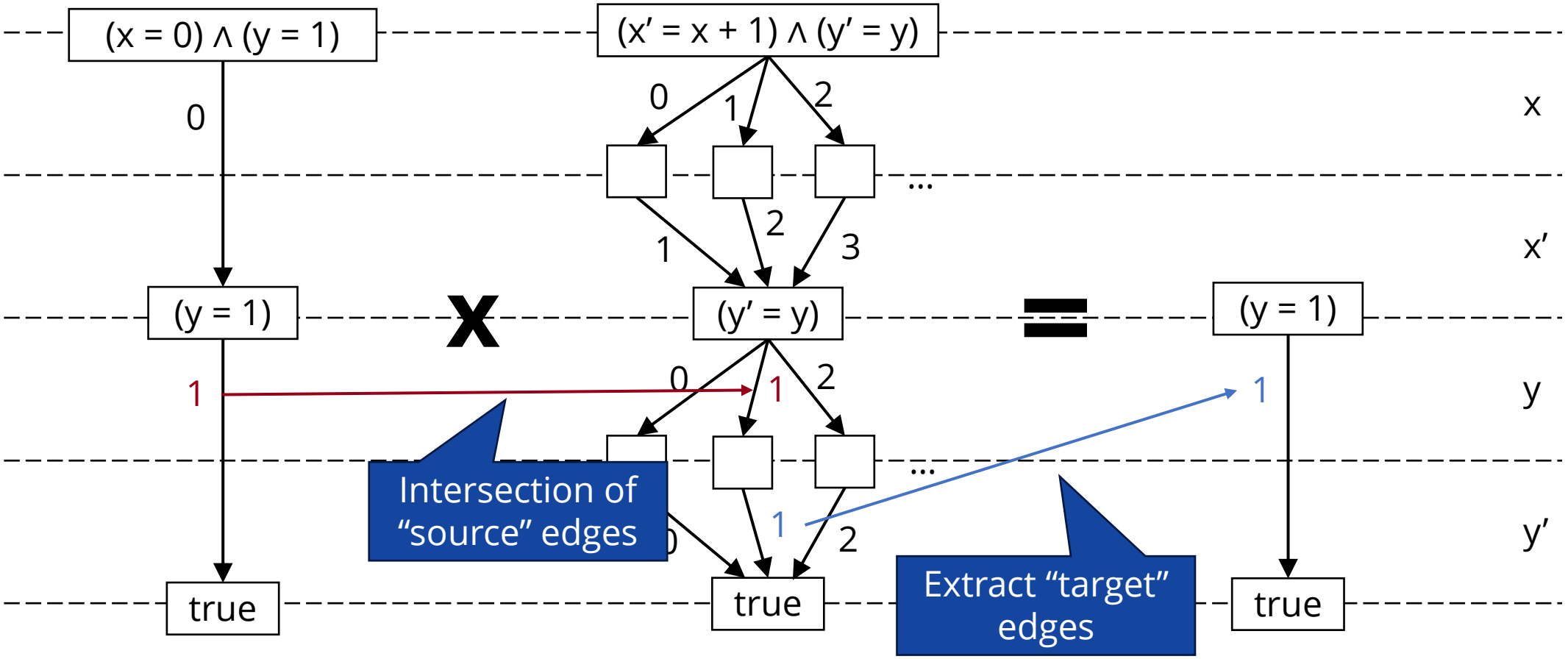

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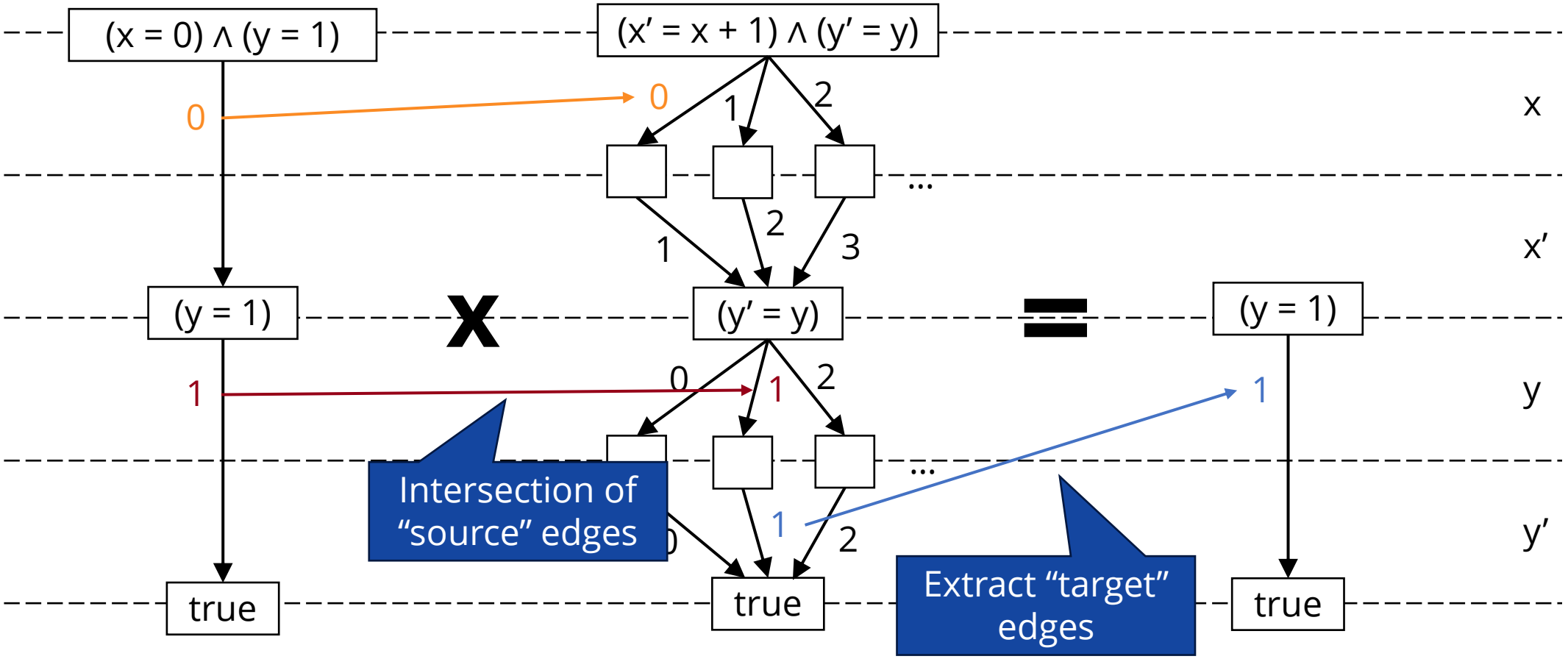

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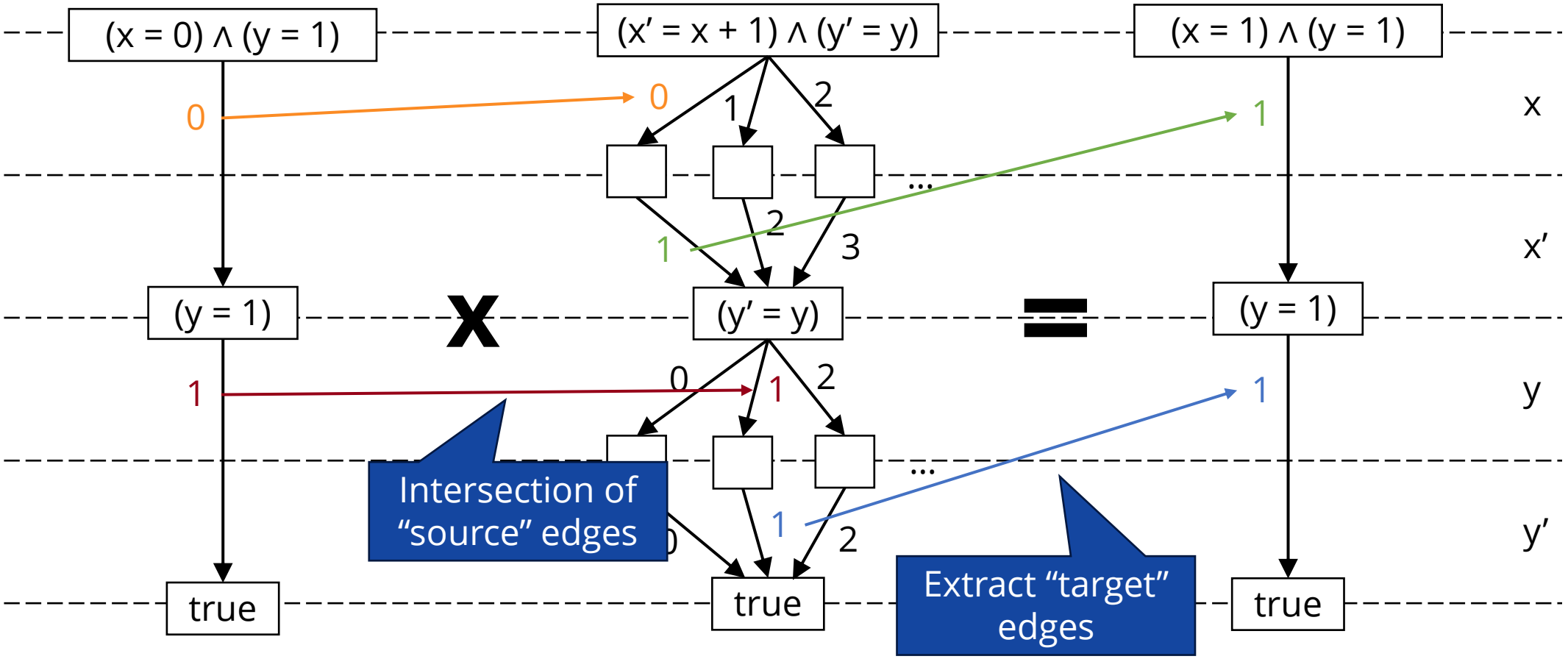

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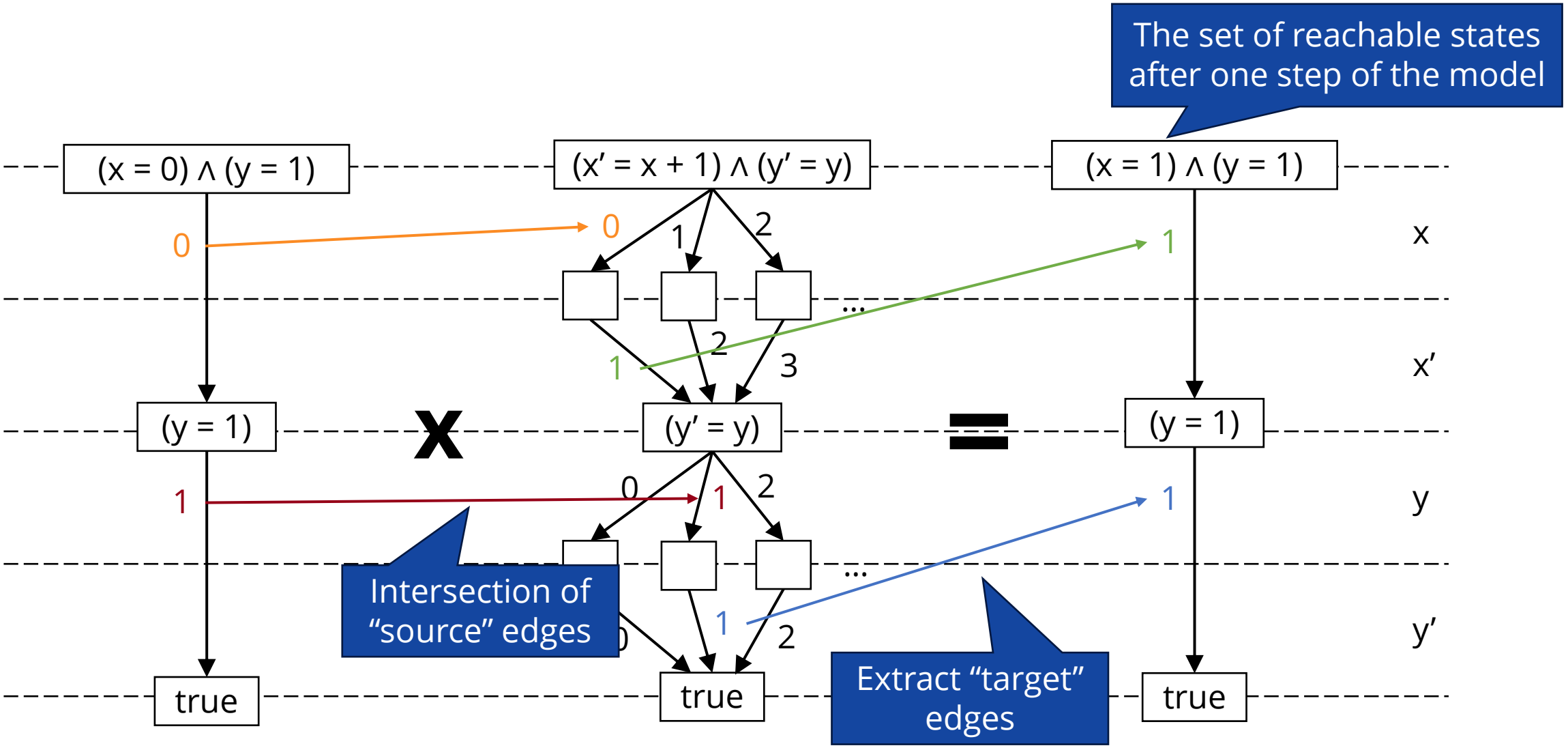

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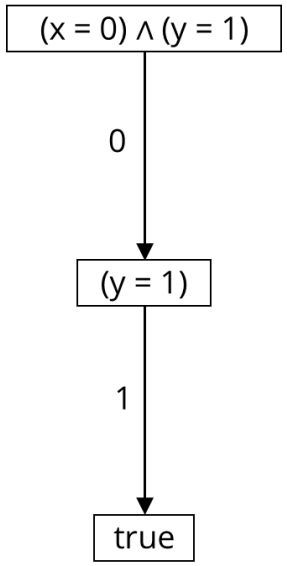

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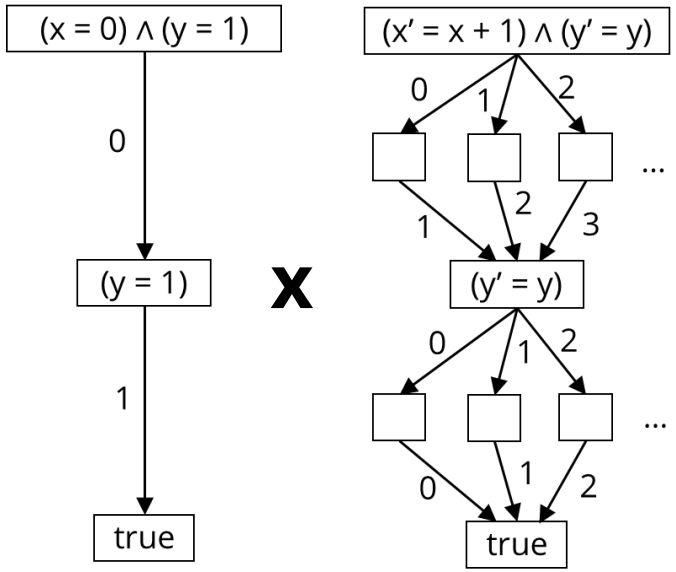

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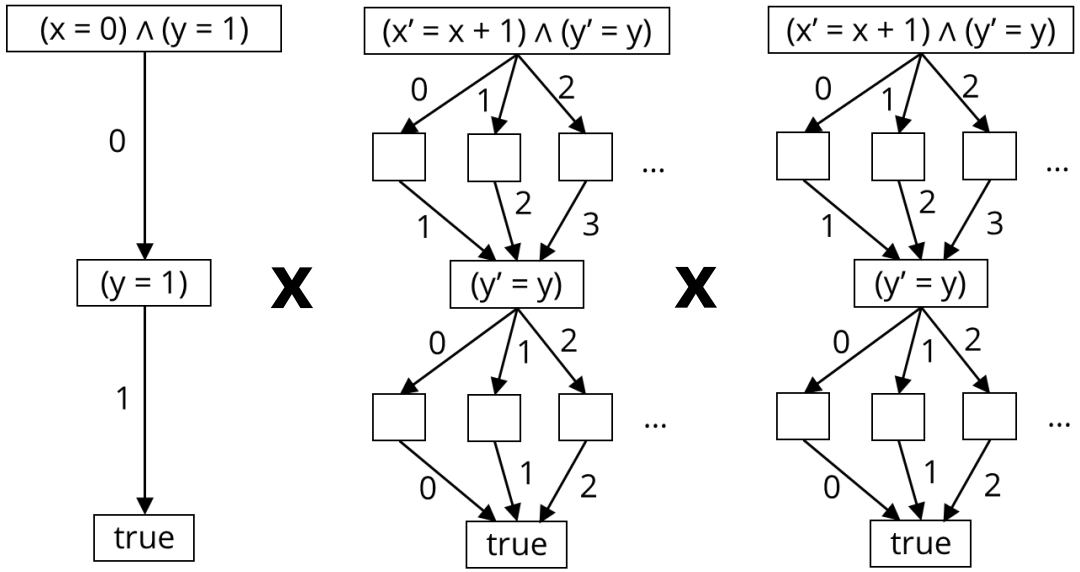

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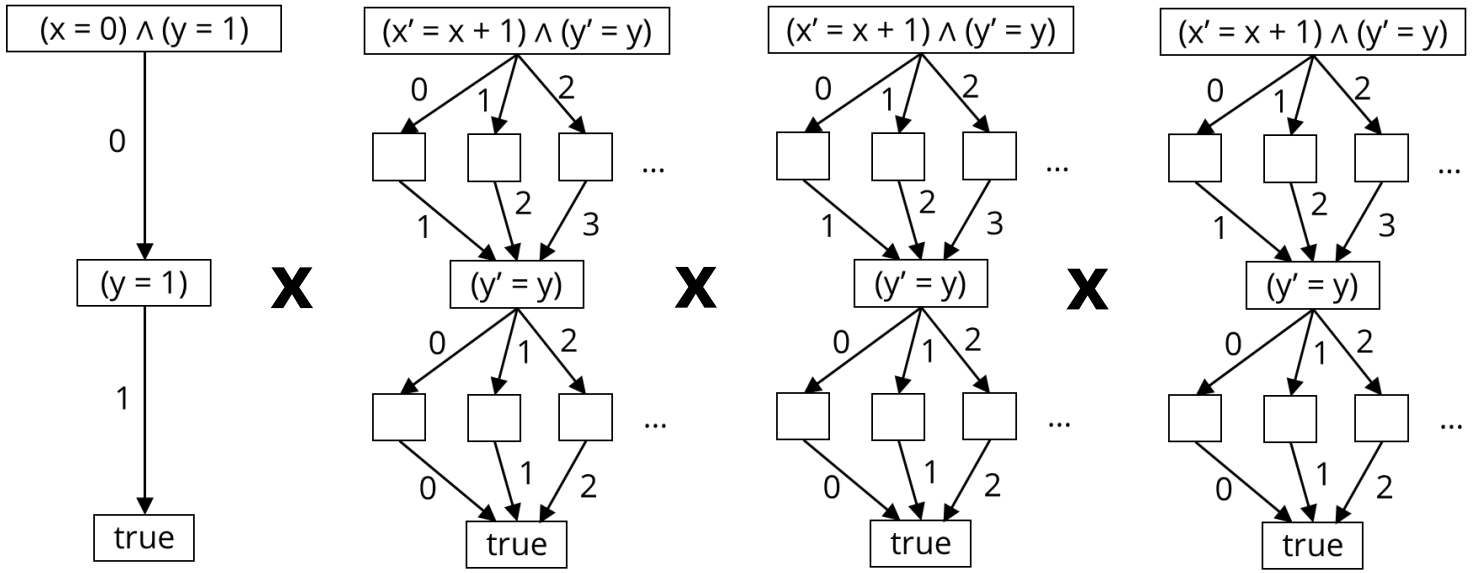

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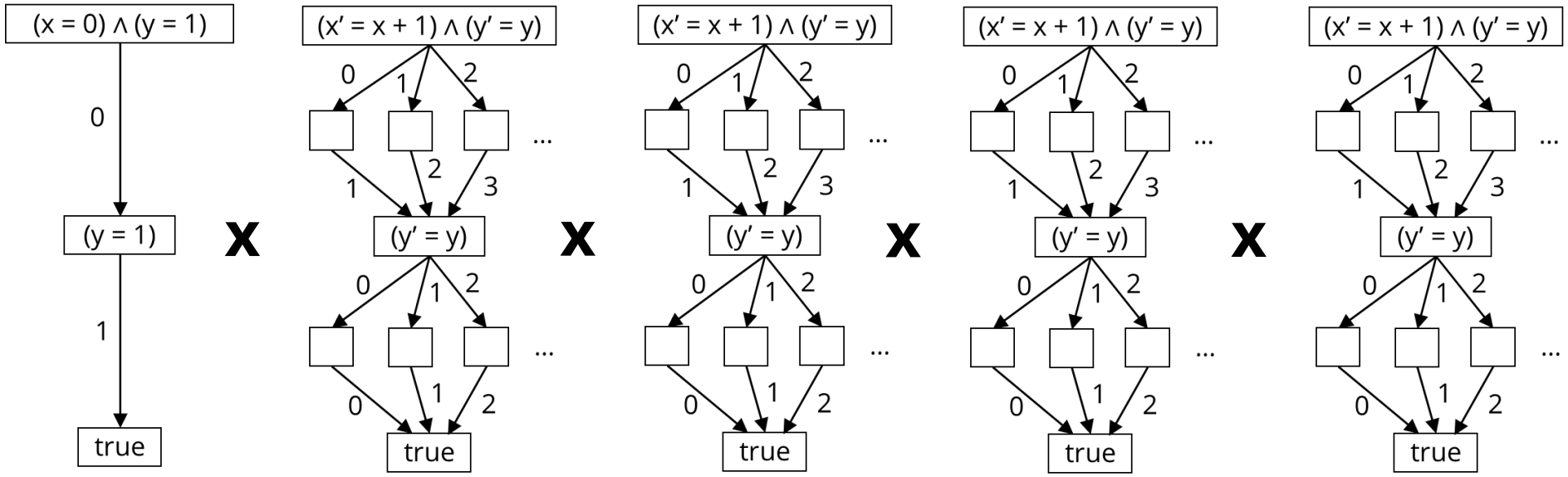

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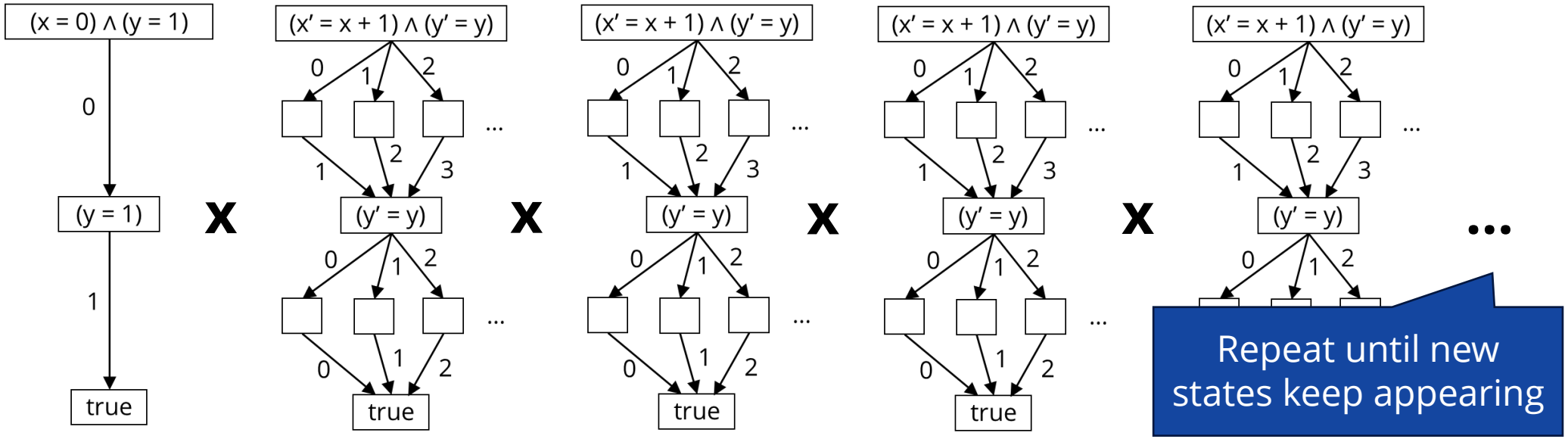

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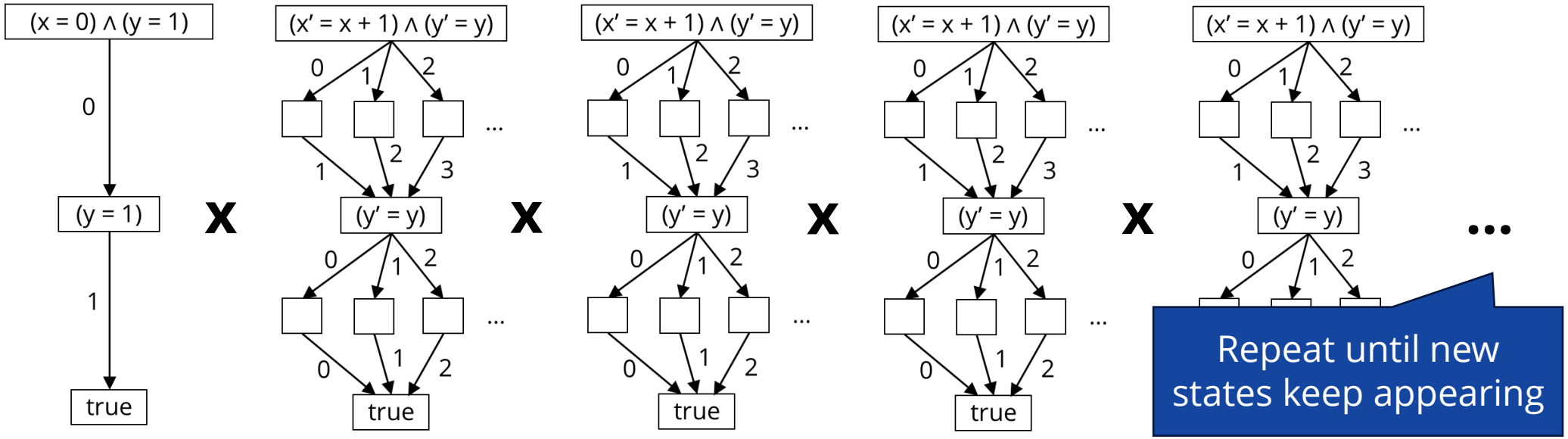

Many possible algorithms: BFS, Saturation

Syntactic vs semantic equivalence

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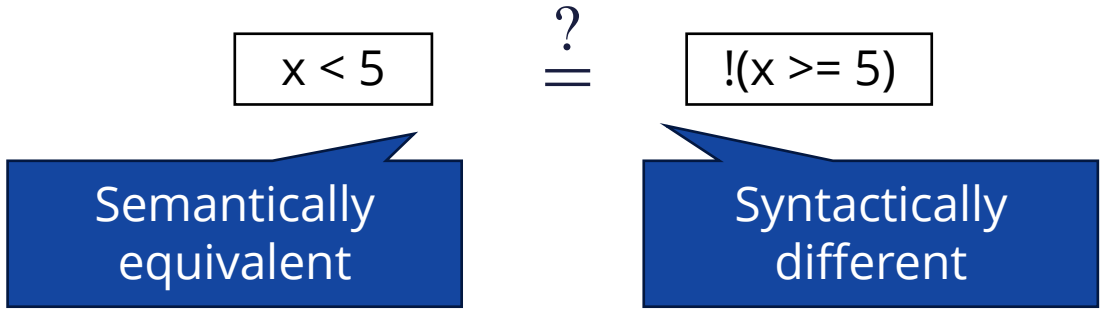

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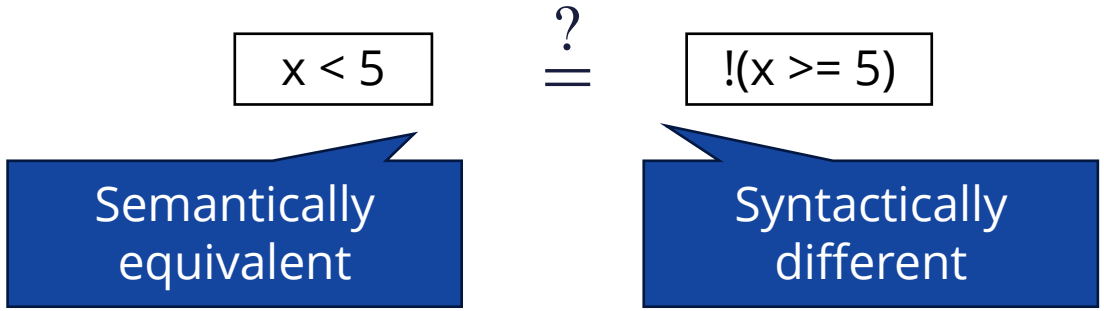

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Syntactic vs semantic equivalence

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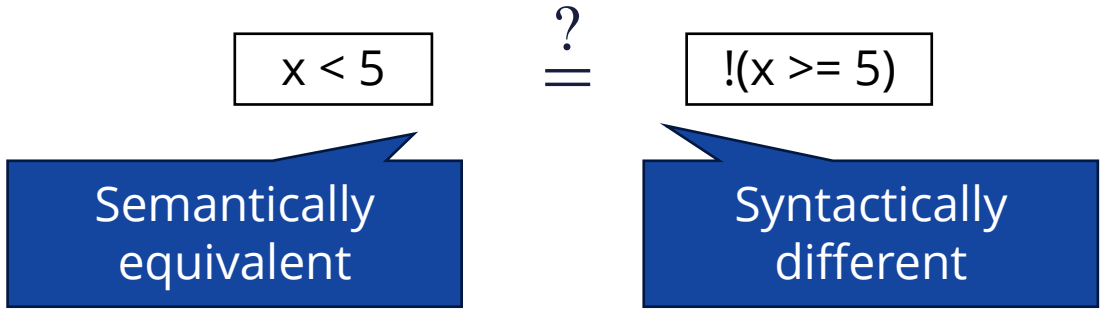

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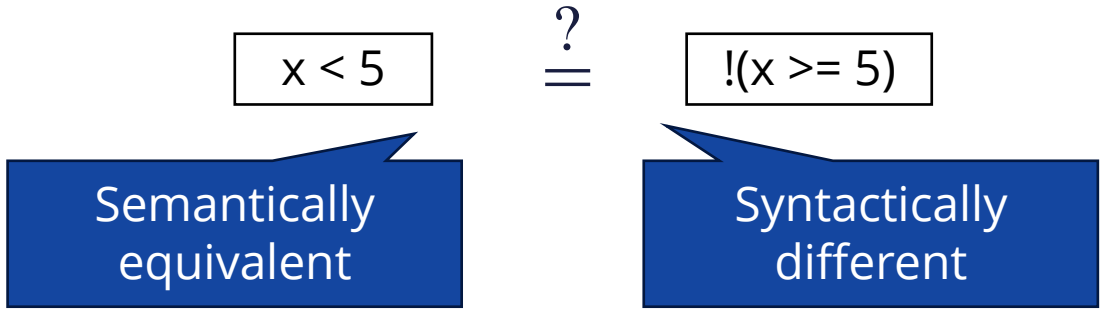

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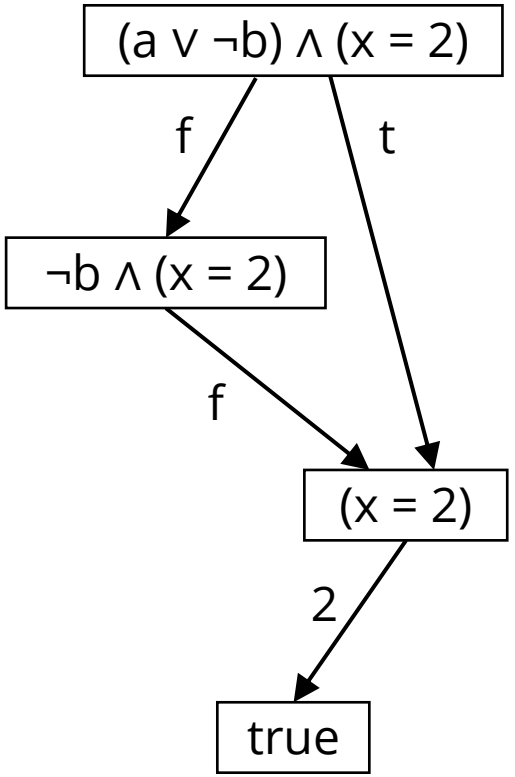
  - Entirely syntactic, no solver used → lightweight

Summary

New data structure: substitution diagram

- Decision diagram structure from SMT formulas
- Lazy evaluation
- Syntactically equivalent nodes are merged
  - Lightweight normal form transformation
- Implemented in the Theta model checker
  - Reachability analysis
  - github.com/ftsrg/theta



Enables the use of **decision-diagram**-based (e.g., saturation) algorithms on **SMT-based** model representations

Evaluation

- How good is our normal form transformation?

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→ 10000 randomly generated transition systems
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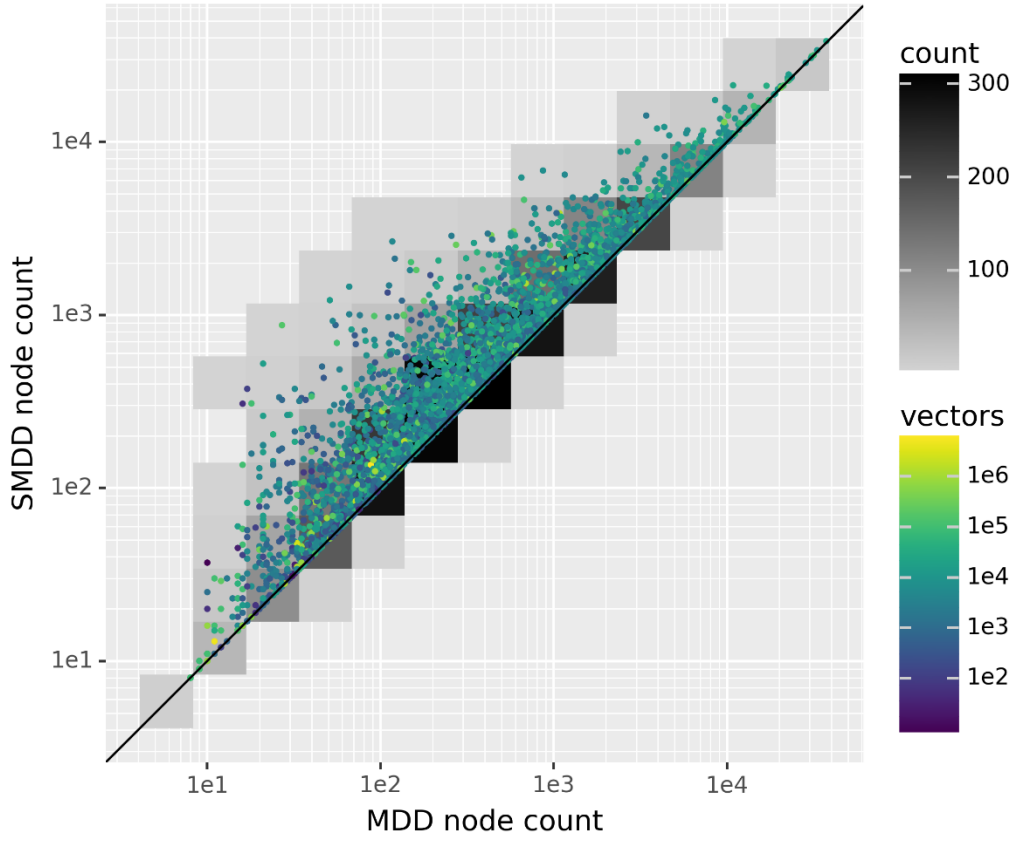
→ Build decision and substitution diagrams from these formulas and calculate node counts

Evaluation

- How good is our normal form transformation?
  - Node count comparison on **3789** randomly generated SMT formulas



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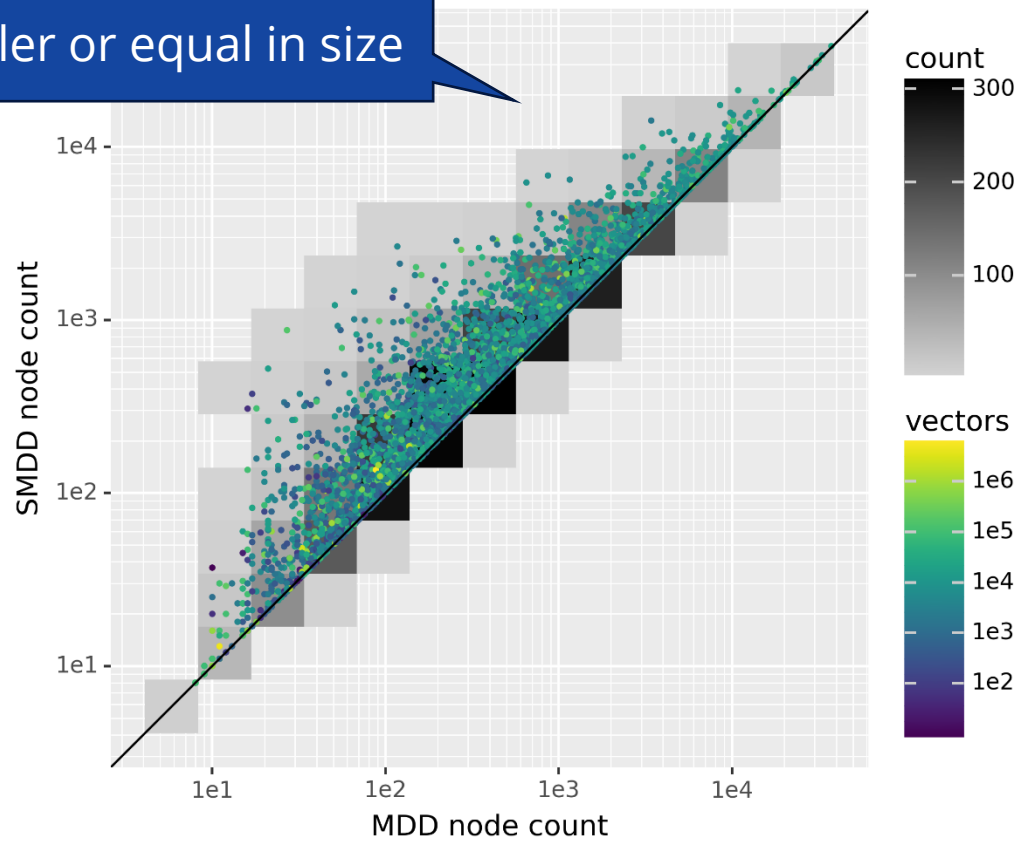
MDD is always smaller or equal in size



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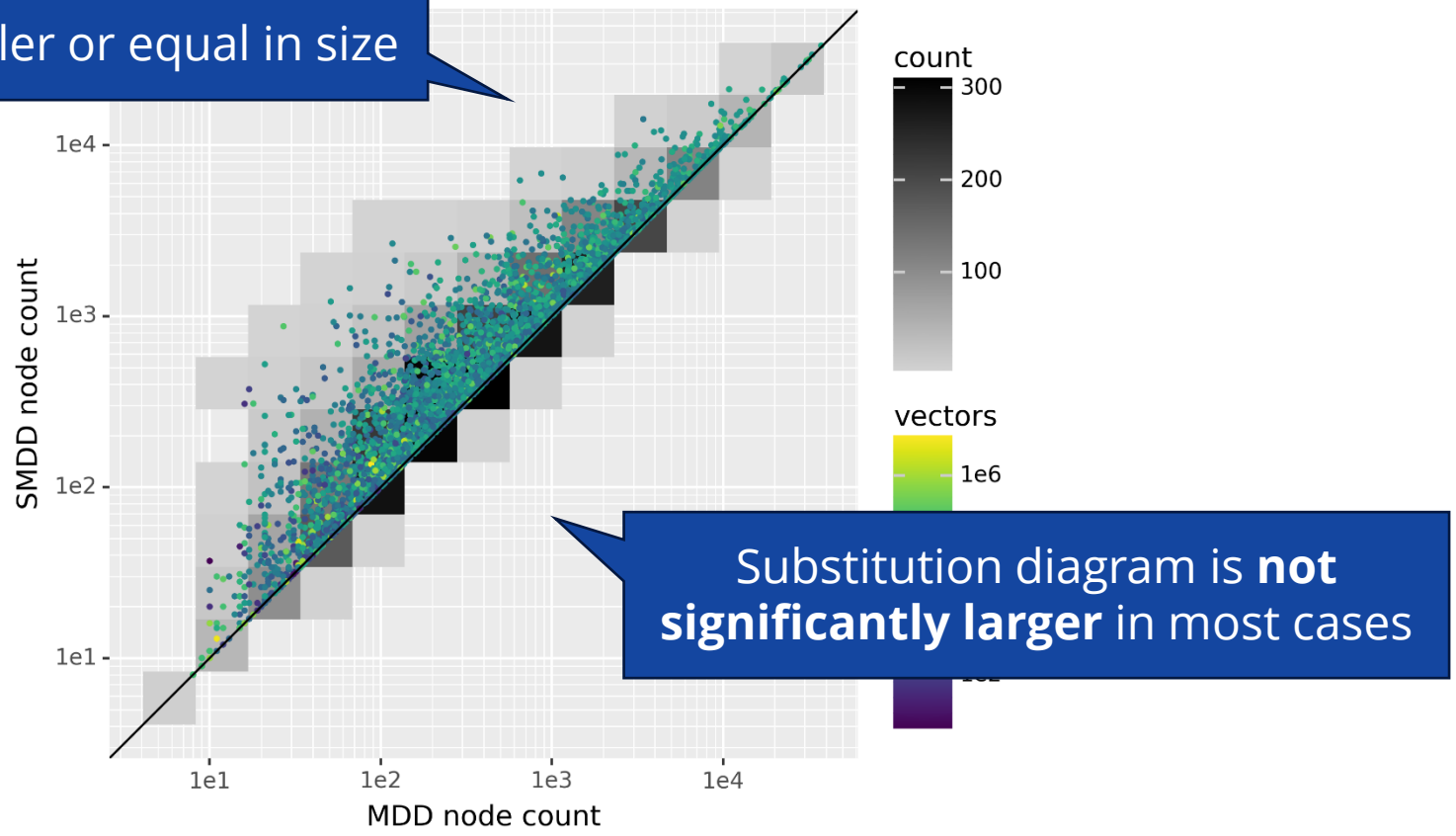

Substitution diagram is **not significantly larger** in most cases